TOPOLOGICAL MEASURE THEORY

A Study of Repleteness and Measure Repleteness

DISSERTATION

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AN ABSTRACT

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Let £ be a lattice of subsets of the abstract set X, and $A(£)$, the algebra generated by £. We define $M(£)$ to be all non-negative, finitely additive, finite valued measures on $A(£)$; $M_{\sigma}(£)$ the measures of $M(£)$ which are σ-smooth on £ and $M_R(£)$ those measure of $M(£)$ which are £-regular. If $\mu \in M_R(£)$ and $\mu \in M_{\sigma}(£)$, we write $\mu \in M_R^{\sigma}(£)$ to represent the countably additive £-regular measures of $M(£)$. $M_{\tau}(£)$ designates those $\mu \in M(£)$ which are τ-smooth on £. If in any of the above definitions, we restrict the measures to be only zero-one valued, the corresponding set is denoted by an I. Thus $I_R(£)$ are the 0-1 valued measures of $M_R(£)$, and $I_R^{\sigma}(£)$ are the 0-1 valued measures of $M_R^{\sigma}(£)$ and so on.

In the first part of this dissertation, we investigate various topological properties associated a lattice £, such as£ normal, regular, etc..., and express these properties in terms of measures from the I's. We also do this for pairs of lattices £$_2 \supset$ £$_1$ thereby extending work of [12], [5], [7] and [2]. These results are the systematically applied to the Wallman Space I_R(£) and lattices the W(£) and τW(£). In particular, conditions for τW(£) to be normal are thoroughly investigated, a similar investigation is carried out for the space I_R^σ(£) with the relative Wallman topology.

In addition, in the first part of the dissertation we give conditions in £ under which $\mu \in M_\sigma$(£) or $\mu \in M_\sigma$(£') (where £' is the complement lattice) or $\mu \in M_\tau$(£') are in M_R(£).

In the second part of the dissertation, the remainders I_R(£)$-X$, and I_R(£)$-I_R^\sigma$(£) for a disjunctive lattice £ are thoroughly investigated and properties in £ such as repleteness, Lindelöff are expressed in terms of these remainders. This work extends that of [3]. Also, special measures of M_R^σ(£) such as strongly σ-additive and strongly τ-additive are introduced, and characterized in terms of induced measures on I_R^σ(£). There behavior under repleteness of £ is also investigated, and new conditions for the repleteness of £ are obtained.

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