

André Platzer  
Geoff Sutcliffe (Eds.)

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# Automated Deduction – CADE 28

28th International Conference on Automated Deduction  
Virtual Event, July 12–15, 2021  
Proceedings

# CADE-28

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# Automated Deduction – CADE 28

28th International Conference on Automated Deduction  
Virtual Event, July 12–15, 2021  
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# Preface

This volume contains the proceedings of the 28th International Conference on Automated Deduction (CADE-28). CADE is the major forum for the presentation of research in all aspects of automated deduction, including foundations, applications, implementations, and practical experience. CADE-28 was hosted by Carnegie Mellon University, Pittsburgh, USA, 11–16 July 2021, but held online due to the COVID-19 pandemic. CADE-28 emphasized the breadth of topics that are of interest, including applications in and beyond STEM, and the use/contribution of automated deduction in AI.

The Program Committee (PC) accepted 36 papers (29 full papers and 7 system descriptions) out of 76 submissions (59 full papers, 4 short papers, and 13 system descriptions). Each submission was reviewed by at least three Program Committee members or their external reviewers. The criteria for evaluation were originality and significance, technical quality, comparison with related work, quality of presentation, and reproducibility of experiments.

The program of the conference included four invited talks:

- Liron Cohen (Ben-Gurion University, Israel): “Non-well-founded Deduction for Induction and Coinduction”
- Guido Governatori (CSIRO, Australia): “Computational Law: Automated Reasoning in the Legal Domain”
- Mooly Sagiv (Tel Aviv University, Israel): “Formal Reasoning about Decentralized Financial Applications”
- Markus Rabe (Google, USA): “What are the Limits of Neural Networks for Automated Reasoning?”

The conference hosted several workshops, tutorials, and competitions:

- Workshop: 10th International Workshop on Theorem Proving Components for Educational Software.
- Workshop: Proof eXchange for Theorem Proving.
- Workshop: Parallel and Distributed Automated Reasoning.
- Workshop: 17th International Workshop on Termination.
- Workshop: Logical Frameworks and Meta-Languages - Theory and Practice.
- Workshop: 3rd International Workshop on Automated Reasoning: Challenges, Applications, Directions, Exemplary Achievements.
- Tutorial: Program Validation and Verification in PVS. Paolo Masci (NIA), Mariano Moscato (NIA), César Munoz (NASA), Aaron Dutle (NASA), and Tanner Slagel (NASA).
- Tutorial: Practice of First-Order Reasoning. Stephan Schulz (DHBW), Adam Pease (Articulate Software), and Geoff Sutcliffe (University of Miami).
- Tutorial: Learning to Prove: Machine Learning for Better SAT and QSAT Solvers. Sean Holden (University of Cambridge).

- Tutorial: Proof-Theoretical Analysis of Non-Fregean Logic. Szymon Chlebowski, Marta Gawek, Dorota Leszczyńska-Jasion, and Agata Tomczyk (Adam Mickiewicz University).
- Competition: 28th CADE ATP System Competition. Geoff Sutcliffe (University of Miami) and Martin Desharnais (Vrije Universiteit Amsterdam).
- Competition: Termination Competition 2021. Albert Rubio (UPC Barcelona) and Akihisa Yamada (AIST Tsukuba).

In addition to the best paper awards, three CADE awards were presented at the conference:

- The Herbrand Award for Distinguished Contributions to Automated Reasoning (for 2020 and 2021).
- The Thoralf Skolem Awards for CADE papers that have passed the test of time by being the most influential papers in the field, for papers from CADE-5 (1980), CADE-11 (1992), CADE-17 (2000), and CADE-23 (2011).
- The (newly established) Bill McCune PhD Award for a PhD thesis' substantive contributions to the field of Automated Reasoning.

Thanks go to the many people without whom the conference would not have been possible - the authors, participants, invited speakers, members of the PC and their subreviewers, conference chairs, local organizers, the workshop/tutorial/competitions chair, the publicity chair, the CADE trustees, the board of the Association for Automated Reasoning, the staff at Springer, and the EasyChair team. CADE-28 gratefully received support from the Automated Reasoning Group at Amazon Web Services, The Journal of Artificial Intelligence, Imandra Inc., and Springer.

July 2021

André Platzer  
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