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Alexander M. Kasprzyk  
Benjamin Nill *Editors*

# Interactions with Lattice Polytopes

Magdeburg, Germany, September 2017

 Springer

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# Preface

This volume contains original research and survey articles highlighting interdisciplinary connections between a diverse range of topics. The common points of interest are lattice polytopes. Lattice polytopes are fundamental combinatorial objects—convex polytopes whose vertices have integer coordinates—with many beautiful and deep connections across modern mathematics. Topics considered include: algebraic geometry, mirror symmetry, symplectic geometry, discrete geometry, the geometry of numbers, and algebraic combinatorics.

The study of lattice polytopes continues to open up fertile and unforeseen interactions. In order to enhance this exchange of ideas, the workshop *Interactions with Lattice Polytopes* took place 14–16 September, 2017, at the Otto-von-Guericke-Universität Magdeburg, Germany. There were 15 talks given by world-leading experts from several different backgrounds, elaborating upon the theme of applications of lattice polytopes. Many of the presented results can be found in this volume.

Contributions to this volume contain original as well as expository research articles that illustrate some of the varied topical approaches and settings where lattice polytopes play an important role. This volume should be particularly beneficial to researchers and graduate students interested in learning more about the multifaceted use of lattice polytopes across a broad range of active research areas.

This book relies deeply on the enthusiasm and engagement of the diverse and collegial lattice polytope community. We are extremely grateful to the contributors for their high-quality articles, and to the anonymous referees for their careful work. We would like to express our gratitude to everyone involved for their patience and assistance. We are also thankful for logistical and financial support from: the Otto-von-Guericke-Universität Magdeburg; the Research Training Group Mathematical Complexity Reduction, funded by the Deutsche Forschungsgemeinschaft (DFG, German Research Foundation)—314838170, GRK 2297 MathCoRe;

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