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Haesung Lee · Wilhelm Stannat · Gerald Trutnau

Analytic Theory of Itô-Stochastic Differential Equations with Non-smooth Coefficients

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Haesung Lee
Department of Mathematics and Computer Science
Korea Science Academy of KAIST
Busan, Korea (Republic of)

Wilhelm Stannat
Institut für Mathematik
Technische Universität Berlin
Berlin, Germany

Gerald Trutnau
Department of Mathematical Sciences
and Research Institute of Mathematics
Seoul National University
Seoul, Korea (Republic of)

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