



Rinaldo B. Schinazi

Probability with Statistical Applications

Third Edition

 Birkhäuser

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Preface to the Third Edition

This edition involves a major reorganization of the book. I have split the material into many more chapters. Many chapters are now largely independent, and this allows for more flexible teaching. The separation between discrete and continuous distributions is sharper.

The style has been changed as well. In particular, there are no longer boxed formulas. In my experience, students focus too much on the formula and try to apply it without noticing differences between problems on the same topic. What has not changed is my teaching philosophy. For each topic, I concentrate on few critical concepts, and I teach them by using many examples.

I have used the first 14 chapters for a first course in probability with statistical applications. This covers all the classical discrete probability distributions and a few continuous distributions (uniform, exponential, and normal). Confidence intervals and several statistical tests are also covered.

I have used some of the material covered in Chaps. 15 through 24 for a second course in probability. Starting with Chap. 15, the mathematical level of the book increases. Calculus of one and several variables is assumed to be known. The level of abstraction is also higher. For instance, we use cumulative distribution functions and moment generating functions. We introduce the classical continuous distributions as transformations of basic distributions. We study joint continuous distributions, covariance, and correlation. We also provide several applications.

The last part of the book is devoted to mathematical statistics. Starting in Chap. 25, we cover estimation (method of moments, maximum likelihood, and Bayes' estimation) and comparison of estimators. I have used material from Chaps. 25 through 29 (supplemented by some previous chapters) for a course in mathematical statistics.

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