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Perturbed Semi-Markov Type Processes II

Ergodic Theorems for Multi-Alternating Regenerative Processes

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ISBN 978-3-030-92398-3 ISBN 978-3-030-92399-0 (eBook)
https://doi.org/10.1007/978-3-030-92399-0

Mathematics Subject Classification: 60J10, 60J22, 60J27, 60K05, 60K15, 60K20, 65C40

This Springer imprint is published by the registered company Springer Nature Switzerland AG
The registered company address is: Gewerbestrasse 11, 6330 Cham, Switzerland

Preface

This book is the second volume of two-volume monograph devoted to the study of limit and ergodic theorems for regularly and singularly perturbed Markov chains, semi-Markov processes, and alternating regenerative processes with semi-Markov modulation.

The second volume presents new super-long, long, and short time ergodic theorems for perturbed alternating regenerative processes and multi-alternating regenerative processes modulated by regularly and singularly perturbed finite semi-Markov processes. These theorems describe the asymptotic behaviour of distributions $\mathsf{P}\{\xi_\varepsilon(t) \in A\}$ as time $t \to \infty$, and the transition characteristics of perturbed multi-alternating regenerative processes $\xi_\varepsilon(t)$ converge to some limiting transition characteristics as the perturbation parameter $\varepsilon \to 0$. Such "individual" ergodic theorems differ from ergodic theorems, such as the laws of large numbers for random averages and related ergodic theorems for the expectations of such averages, based on the use of stationarity arguments. The theorems presented in the book are based on the use of renewal type arguments in combination with new asymptotic recurrent algorithms of phase space reduction for perturbed multi-alternating regenerative processes.

In the first volume, we present limit theorems on the weak convergence of distributions and the convergence of expectations of hitting times for regularly and singularly perturbed semi-Markov processes, based on algorithms of phase space reduction for perturbed semi-Markov processes. These results play an important role in obtaining the ergodic theorems presented in the second volume.

Models of perturbed Markov chains and semi-Markov processes, in particular for the most complex cases of so-called singularly perturbed processes, attracted the attention of researchers in the middle of the twentieth century. Interest to these models has been stimulated by applications to control and queuing systems, information networks, epidemic models, and models of mathematical genetics and population dynamics. As a rule, Markov-type processes with singular perturbations arise as a natural tool for the mathematical analysis of multi-component systems with weakly interacting components.

The most deeply investigated are cases of Markov chains and semi-Markov processes with finite phase spaces. There is a huge bibliography of works that contain ergodic theorems and related limit theorems for hitting times for perturbed finite Markov chains and semi-Markov type processes. However, the theory of such ergodic and limit theorems is still far from completion. The results presented in the book, I hope, confirm this point of view well.

The second volume includes an introduction, 12 chapters grouped in two parts, and three appendices.

In Introduction (Chap. 1), simple examples, models of stochastic processes, conditions, and results are presented in an informal way. It also provides chapter-by-chapter content and additional information for potential readers.

Part I (Chaps. 2–9) presents ergodic theorems for perturbed regenerative processes with regenerative lifetimes, as well as the results of a detailed analysis and complete classification of super-long, long, and short time ergodic theorems for regularly, singularly, or super-singularly perturbed alternating regenerative processes modulated by two-state semi-Markov processes.

Part II (Chaps. 10–13) presents super-long, long, and short time ergodic theorems for regularly or singularly perturbed multi-alternating regenerative processes modulated by finite semi-Markov processes. This is done using new asymptotic recurrent algorithms for time-space aggregation of regenerative times and phase space reduction for modulating semi-Markov processes, which allow us to reduce the above ergodic theorems to the corresponding ergodic theorems for perturbed alternating regenerative processes presented in Part I.

Appendix A presents generalisations of the classical renewal theorem to a model of the perturbed renewal equation.

Appendix B presents some additional asymptotic results commonly used throughout the book.

Appendix C contains some methodological and bibliographical notes, as well as comments on the new results presented in the book and some new problems for future research.

I hope that the publication of this new book on asymptotic problems for perturbed stochastic processes will be a useful contribution to the ongoing intensive research in this area. In addition to being used for research and reference purposes, the book can also be used in special courses on this topic and as additional reading to general courses on stochastic processes. In this regard, it can be useful for both specialists as well as doctoral and senior students.

I would also like to thank my colleagues at the Department of Mathematics, Stockholm University and at the Division of Mathematics and Physics, School of Education, Culture and Communication, Mälardalen University for creating an inspiring research environment and a friendly atmosphere, which stimulated my work.

Stockholm, Sweden Dmitrii Silvestrov
October 2021

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