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Farrukh Mukhamedov
Ahmed Al Rawashdeh *Editors*

Infinite Dimensional Analysis, Quantum Probability and Applications

QP41 Conference, Al Ain, UAE, March 28–April 1, 2021

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Preface

The fact that Quantum Theory (QT) is not only a new mechanics but also a new probability was clear from the very beginning of this discipline, in the first quarter of the twentieth century. While the systematic comparison between classical and quantum mechanics has always been present in QT, the systematic investigation of the probabilistic aspects of quantum theory began in the early 1970s. From that time to the present day, the field of quantum probability (QP) has grown enormously.

At the beginning, the QP conferences were a meeting point for a small group of people coming from different countries who had a clear intuition of the potentialities of this new field and were united by the goal of promoting its development. Since the interactions between probability and QT arise in a multiplicity of scientific levels, from philosophical problems concerning the foundations of probability and physics, to the solution of specific problems arising from physics and the development of new mathematical techniques to deal with them, the multidisciplinarity of the field was one of the characteristics of QP since its early developments. Practically, no other mathematical discipline, after the Second World War, has given so many conceptually deep and innovative contributions to so many branches of science. From the mathematical explanation of the old quantum paradoxes, which the philosopher Karl Popper labeled *the great quantum muddle*, to the development of the first deductive (as opposed to phenomenological) theory of quantum transport in open systems, to the emerging applications of non-Kolmogorovian models in sciences different from physics, to the discovery of the natural emergence, from the combination of classical probability with the classical theory of orthogonal polynomials, of *general quantization rules* which explain the probabilistic roots of Heisenberg commutation relations and explains why future *interacting quantum theories* cannot be based on such relations. Parallel to this challenge to physics, the same discovery poses an optimistic challenge to classical probability and statistics: to learn how to interpret and put to use the quantum aspects of classical random variables, processes, and fields which are ubiquitous outside quantum physics.

If, from deep conceptual innovations, we pass to relevant technical progresses (which often are the basis of conceptual innovations), then the list of contributions of QP becomes considerably longer. From the emergence of quantum Markov chains, on

which nowadays the most sophisticated numerical technique for the approximation of ground states of quantum Hamiltonians is based, to quantum stochastic calculus, without which no deductive theory of quantum transport could exist, to white noise hamiltonian calculus, unifying the stochastic differential equation approach used in mathematics with the distribution (white noise) approach used in physics, to new objects in operator theory like the canonical conditional expectation of a state, to the introduction of local equilibrium fields, to the universality of the non-crossing diagrams in the stochastic limit of strongly non-linear interactions (versus the universality of the gaussian diagrams in usual quantum field models), to the connections between random matrices and free independence or between the adjacency matrices of graphs and various forms of independence (monotone, boolean, ...), to the deep connections between interacting Fock spaces and classical orthogonal polynomials, to the first deduction of the natural emergence of Hilbert modules in QT (notice that the great mathematical development of Hilbert's module theory in the last three decades occurred after this discovery). The above list is far from exhaustive, but it is sufficient to show that QP abundantly passes the only criterium that distinguishes deep innovations from individual, although important, results, namely, the capacity to give essential contributions to different fields of science, and in particular of mathematics.

Clearly, given the width of the scientific front, very few people could contribute to all these developments. However, quite a number of people gave relevant contributions to several of them and this fact, on one side, played a relevant role in keeping up the initial multidisciplinary inspiration, on the other side, confirms the historical fact that every deep innovation in science is a collective phenomenon, involving several groups of scientists in several different countries.

Now, in almost 50 years of life, QP has reached maturity and begins to face the problems intrinsic to every mature mathematical discipline, namely, the conflict between specialization and multidisciplinarity. This induced a gradual change in the purpose of the QP conferences: from meeting points of a few researchers with strong interactions among themselves to moment of interaction and mutual confrontation among groups dealing with problems that, although stemming from a common root, employ different and sometimes very mathematical techniques for different purposes. With the explosion of Quantum Technology, the promotion of applications to industrial problems is going to become one of the main goals of the conferences.

The 41st International Conference on Quantum Probability and Related Topics, was held at the UAE University, Al Ain city-UAE from March 28 to April 1, 2021. The conference was dedicated to the memory of Robin Hudson, one of the pioneers of quantum probability, who passed away on January 12, 2021. Unfortunately, just a few months before the opening of the conference, two other pioneers of quantum probability passed away: Andrzej Kossakowski, famous for his contributions to the theory of open quantum systems, and Wilhelm von Waldenfels author of the first universal quantum central limit theorems and who introduced co-algebra techniques in QP long before the emergence of quantum groups. During the conference, a commemorative speech on Kossakowski who passed away on February 1, 2021 was delivered by Dariusz Chruscinski and one in memory of Wilhelm von Waldenfels was given

by Luigi Accardi. The latter will appear in a special issue of IDAQP 2022 dedicated to both Robin Hudson and Wilhelm von Waldenfels, good friends in life and united by an almost simultaneous death.

As usual, this conference covered recent developments in Quantum Probability and Infinite Dimensional Analysis, with applications to Mathematical Physics, Quantum Information Theory, and other related fields.

The multidisciplinary nature of Quantum Probability is well reflected by the structure of the volume, which is divided into six parts:

I. Quantum Probability Methods
II. Quantum Information Methods
III. Quantum Dynamical Systems
IV. Infinite Dimensional Analysis
V. Operator Algebras
VI. Stochastic Operators

This wide spectrum of topics shows that the original goal of the organizers was fully realized. Several new results, some of which are of great depth, were presented at the conference and show the vitality of the field and its continual propensity to interact with different disciplines and to cover a wide spectrum of interests.

The 42nd QP conference will take place in the last week of January 2022 in Bangalore thus continuing the tradition and confirming the international reality of the QP community (the past ten QP conferences have taken place always in different nations, covering four continents). Readers of the volume are anticipated to be graduate students and research mathematicians interested in functional analysis, classical and quantum probability, operator algebras, Lie algebras, mathematical physics, and quantum information, quantum technology.

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