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Markov Processes and Quantum Theory

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Preface

Quantum Theory is more than one hundred years old, but its mathematical foundations have been gradually clarified only in the last half-century. We will tell the story in this book. The whole contents of quantum theory will be unified under the name of *theory of stochastic processes*. As a matter of fact, in our view, Quantum Theory is an application of the theory of Markov (stochastic) processes to the analysis of the motion of small particles in physics (such as electrons and photons).

It is correctly considered that quantum theory is radically different from the classical theory (i.e., Newton's mechanics). In fact, quantum theory is obtained by means of the quantization of the classical theory. Quantization means that we replace classical physical quantities with operators and subject them to the so-called commutation relations. This procedure itself is the core of the Born-Heisenberg quantum mechanics and is responsible for the successes of quantum theory. But why we must proceed in this way was never clearly explained. It has been accepted that success itself justifies the method. However, this incomprehensible so-called quantization is a technical matter, and quantum theory should be understood in the context of the theory of stochastic processes, as will argue in the sequel.

In this book we adopt the theory of Markov processes as a mathematical foundation of quantum theory instead of quantization.

In the conventional quantum theory, it has been *implicitly* assumed that the paths of the motion of a particle are smooth, like in the classical Newtonian mechanics. In this book we assume that, on the contrary, the paths of the motion of a small particle are not smooth; rather, the particle performs a random zigzag motion. We will explain and prove that *the theory of random motion of small particles covers all of what has been called quantum theory*. In the conventional quantum theory the notion of random motion of small particles is somehow neglected and has never been considered seriously. We will attempt to make as clear as possible the fact that quantum theory is precisely the theory of random motion. To develop the new quantum theory, we will extend slightly the conventional theory of Markov processes, and we will do it step by step in each chapter, as required.

To make things clearer, let us quickly look back at the historical development of *quantum theory* . Here we call the whole discussion on the motion of electrons and photons (or other small particles) *quantum theory.* The origin of this theory is Planck's *quantum hypothesis*, formulated in 1900. In deriving his heat radiation formula, Planck postulated that light energy can only be emitted and absorbed in discrete bundles, called quanta. However, what this actually means was not clear until Einstein's particle theory of light appeared (Einstein (1905, a)). Particles of light are then called photons. The physical objects that quantum theory treats are small particles such as electrons and photons.

If so, we must then determine how such small particles move under the action of external forces. In the conventional quantum theory, discussions on the trajectories of small (quantum) particles have been neglected. In fact, if you look for discussions on the trajectories of small (quantum) particles in the available quantum theory books, you will certainly be disappointed, because you will find nothing. You will find there mainly methods for computing energy values. In fact, the existing so-called theory of quantum mechanics offers no possibility of computing trajectories of the motion of electrons (small particles). One might argue that the Schrödinger equation tells us about the motion of electrons under external forces (potentials). But how? As a matter of fact, the Schrödinger equation is an equation for an "amplitude of probability" and does not give information on trajectories of electrons. For this we must resort to our imagination. This is the only way one proceeds to understand the so-called quantum mechanics.

In this book, we will develop **a theory that enables us to compute and describe the motion, namely, trajectories of electrons (of small particles), and not only to compute energy.** In fact, small particles such as electrons perform a Brownian motion, which is an unavoidable, intrinsic motion of small particles, because it is caused by vacuum noise. Hence, the paths of small particles are not differentiable everywhere. We will accept this fact and call it **random hypothesis**.

As a consequence, Newton's classical equation of motion is not applicable to the random motion, and we need a **new equation of motion** which can be applied to particles performing random motions.

We call the theory of random motion of particles that is based on these new equations of motion **mechanics of random motion**, i.e., a **new quantum theory**. It is an elaboration of quantum theory in terms of the theory of Markov processes. This is the main theme of the present book. However, to formulate the new equations of motion we must extend the conventional theory of Markov processes.

If one applies the *mechanics of random motion* to the physical phenomena that have been called *mysteries of quantum mechanics*, the mystery is removed. This kind of quantum mechanical phenomena will be explained clearly in the theory of random motion. Moreover, problems that could not be handled in quantum mechanics will be brought under the clear and bright light of the theory of random motion and solved unequivocally, because we can now compute accurate trajectories (paths) of the motion of electrons and see how electrons move under the influence of external forces.

Now, if one looks at the history of quantum theory and the history of the theory of stochastic processes, one finds that they both started at the beginning of the twentieth century, and developed in parallel independently, except for Schrödinger's analysis in 1931. Nevertheless, the two theories were intimately related, although most of people did not recognize this clearly.

In quantum theory, the Schrödinger equation was discovered in 1926. Schrödinger investigated the Brownian motion which is symmetric with respect to time reversal (Schrödinger (1931)) in order to deepen his understanding of the Schrödinger equation. On the other hand, in the theory of stochastic processes, Kolmogorov laid the foundations of probability theory and discovered the equation of diffusion processes that today bears his name (Kolmogoroff (1931, 1933)). The two equations are remarkably similarity.

In fact, although this is not well known, Kolmogorov's elaboration of probability theory and Kolmogorov's equation of diffusion processes were strongly influenced by Schrödinger's investigation of the Brownian motion and the Schrödinger equation. Moreover, motivated by Schrödinger's investigation of time reversal of Brownian motion (Schrödinger (1931, 1932)), Kolmogorov discussed the time reversal and the duality of the diffusion processes with respect to the invariant measure (Kolmogoroff (1936, 1937)).

There is another remarkable similarity. The Schrödinger equation contains the first-order time derivative multiplied by the imaginary unit i. On the other hand, Kolmogorov's equation also contains the first-order time derivative, but without the imaginary unit i. Hence, the Schrödinger equation is an equation for complex-valued functions, whereas the Kolmogorov equation is one for real-valued functions. Nevertheless, the two equations describe the same physical phenomena, namely, the random motion of small particles under external forces, although this is not self-evident. In fact, Schrödinger recognized this fact and tried to clarify it in his work (Schrödinger 1931), but that attempt was not so successful. This is a difficult problem. We will analyze Schrödinger's problem and solve it in this book.

In short, we will clarify the intimate relationship between Schrödinger's work and that of Kolmogorov, and unify the two theories into a single theory. This unified theory is

the new quantum theory i.e., **mechanics of random motion.**

In other words, we will argue that **quantum theory is nothing but mechanics of random motion under the influence of external forces.**

Now, as we have explained, in the theory of random motion the **random hypothesis** is an indispensable basic notion. In the quantum mechanics of Born and Heisenberg, however, this recognition of the importance of the random hypothesis is missing. This will be explain further in Chapter 3. (Cf. also Luis de la Peña et al. (2015).)

We will first present in Chapter 1 some aspects of the theory of stochastic processes, in particular, Kolmogorov's equation, and Itô's stochastic differential equations and Itô's formula. This part is, in a sense, a fast introduction to the theory of Markov processes. Then, we will introduce the **equation of motion for stochastic processes**, which is a new notion, and provide mathematical tools to handle it.

We will then discuss the superposition of complex evolution functions of stochastic processes. Superposition induces the so-called entangled random motion. As an application of superposition (the entangled motion), we solve the problem of the double slit experiment, which was one of unsettled problems of quantum mechanics. In quantum mechanics people speak of *interference* to explain the double-slits problem (cf., e.g., Merli P. G., Missiroli G. F., and Pozzi G. (1976), Rosa (2012)). However, it is clear that interference is not possible if we consider an experiment in which we send electrons one by one, with a sufficiently long time separation.

There is one more point deserving attention. As is well known, in the conventional theory of diffusion processes one specifies an initial value for an evolution equation. In our theory of mechanics of random motion, instead, we specify both an initial value *and* a terminal value for an evolution function. We will see that this is one of the most important facts and advantages of our theory.

Chapter 2 is devoted to applications of the theory. We will show that we can follow exact trajectories of electrons moving even in inside atoms. In particular, we will see that Bohr's transition between energy levels is not a jump, but a continuous change of motion described by the Schrödinger equation.

In Chapter 3, we argue that Heisenberg's uncertainty principle is erroneous, and that Einstein's locality holds, contrary to Bell's claim. An example satisfying the locality property is given.

In Chapter 4 we provide the Feynman-Kac and Maruyama-Girsanov formulas, and also explain the time reversal in stochastic processes and its applications.

Chapter 5 introduces the concept of the relative entropy of stochastic processes, and discusses the so-called "propagation of chaos" of Kac as an application in this context.

The creation and annihilation of random particles will be discussed in the framework of the theory of Markov processes in Chapter 6.

The contents of this volume overlaps to some extent with the contents of two other monographs Nagasawa (1993 and 2000). In Nagasawa (1993) emphasis is placed on clarifying the relationships between the Schrödinger equation and diffusion processes. Nagasawa (2000), on the other hand, provides an exposition of the theory of Markov processes and also of time reversal in Markov processes, which plays an essential role in quantum theory of random motion. In fact, the system of equations of motion of quantum particles consists of a pair of equations, one in which time runs forward, and a second one in which time runs backward, so time reversal plays a key role. In the present volume the entire material is systematically presented as a theory of Markov processes in quantum theory.

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