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B. Ramakrishnan
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Modular Forms and Related Topics in Number Theory

Kozhikode, India, December 10–14, 2018

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Preface

This volume includes selected papers presented at the Conference on Number Theory, held during 10–14 December 2018 at the Kerala School of Mathematics (KSoM), Kozhikode, India. More than 30 talks were given at the conference by mathematicians from India and abroad.

These proceedings are being published in honour of Prof. M. Manickam. Professor Manickam did his Ph.D. from Ramakrishna Mission Vivekananda College, Chennai under the guidance of Prof. T. C. Vasudevan in 1990. After finishing his Ph.D., he joined the same college as a faculty member and taught for about 20 years. In the year 2011, he moved to KSoM as Director-in-Charge. He has written about 30 research articles in the area of modular forms of integral and half-integral weights, Jacobi forms and Siegel modular forms. He has mentored many research students and teaching faculty with regard to their research works. He has organised many outreach programmes and workshops for the benefit of researchers working in the field of number theory during his time at KSoM.

We invited articles from all the speakers of the conference and his well-wishers in the community of mathematics. With an overwhelming response, we received 15 contributions for this volume. We convey our heartfelt gratitude to all authors who contributed to the volume. All articles were refereed by experts in the respective field and we take this opportunity to thank them all for their timely help and support. We are sure that the articles in this volume are beneficial to the mathematics community, especially for researchers working in number theory. There are 15 chapters in this volume, out of which 11 works deal with different aspects of modular forms; two chapters are in combinatorial number theory; and one each in algebraic number theory and general mathematics. Below we mention a brief description of articles that appear in this volume:

Chapter "On Vanishing of Hecke Operators" gives a brief survey of known results on Hecke operators acting on the space of cusp forms of integral weight for full modular group. A method of Ronyai and a smart application of Chevalley–Warning and Alon's Combinatorial Nullstellensatz are presented in chapter "On a Polynomial Method of Rónyai in the Study of Zero-Sum Theorems". Chapter "*p*-adic Asai *L*-functions Attached to Bianchi Cusp Forms" deals with establishing

a rationality result for the twisted Asai L-values attached to a Bianchi cusp form and constructing distributions interpolating these L-values. Chapter "Arithmetic Properties of Vector-Valued Siegel Modular Forms" describes how one can get arithmetic properties of vector-valued Siegel modular forms (more precisely: integrality properties of their Fourier coefficients) by combining the doubling method with certain holomorphic differential operators studied by T. Ibukiyama. Some omega results for Fourier coefficients of half-integral weight and Siegel modular forms are discussed in chapter "Omega Results for Fourier Coefficients of Half-Integral Weight and Siegel Modular Forms". In chapter "On Hecke Theory for Hermitian Modular Forms", the Hecke theory for Hermitian modular forms in the sense of Hel Braun for arbitrary class number of the attached imaginary quadratic number field is outlined. Sign changes of coefficients of the powers of the Dedekind eta-function, in particular, the Ramanujan tau function is studied in chapter "Sign Changes of the Ramanujan τ-Function". Chapter "The Central Limit Theorem in Algebra and Number Theory" elucidates the impact of the central limit theorem in number theory and algebra, in particular, probabilistic number theory, Goncharov's theorem, normal number of prime factors of Fourier coefficients of modular forms and probabilistic connections to the Riemann hypothesis. In chapter "Rankin–Cohen Brackets and Identities Among Eigenforms II", a characterisation of eigenforms which are Rankin–Cohen brackets of two quasi-modular eigenforms is studied. Chapter "Rankin–Cohen Type Operators for Hilbert–Jacobi Forms" deals with construction of Rankin–Cohen type differential operators on the space of Hilbert–Jacobi forms. In chapter "Determining Modular Forms of Half-Integral Weight by Central Values of Convolution L-Functions", it is shown that a Hecke eigenform of half-integral weight is uniquely determined by the central values of a family of convolution (Rankin–Selberg) L-functions. By constructing explicit bases for the spaces of modular forms of weight 2, level 48 with different characters, formulas for the number of representations of a positive integer n by certain quaternary quadratic forms are obtained in chapter "On the Number of Representations of a Natural Number by Certain Quaternary Quadratic Forms" along with some discussion on universal quadratic forms. A combinatorial number on the symmetric group is defined and a connection between this number and the Stirling number of first kind is established in chapter "Identities from Partition of the Symmetric Group S_n". In chapter "A Certain Kernel Function for L-Values of Half-Integral Weight Hecke Eigenforms", a non-cusp form of half-integral weight in the Kohnen plus space is derived whose Petersson scalar product with a Hecke eigenform gives the special values of the L-function associated to the Hecke eigenform. Finally, in chapter "On Admissible Set of Primes in Real Quadratic Fields", two simple families of real quadratic fields are considered and a construction of admissible set of primes in these fields is demonstrated.

This proceeding will serve as an important resource for undergraduate/graduate students as well as researchers interested in broad aspects of number theory, modular forms and combinatorics.

We take this opportunity to thank the Kerala State Council for Science, Technology and Environment (KSCSTE) and the National Board for Higher Mathematics (NBHM) for providing financial support for organising the conference. It is our pleasure to thank Mr. Shamim Ahmad for encouraging us to submit the proposal to Springer. Finally, we thank Springer Nature for accepting our proceedings to be published under the series 'Springer Proceedings in Mathematics & Statistics'.

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April 2020

Conference on Number Theory

December 10–14, 2018
Kerala School of Mathematics, Kozhikode
Organisers: T. Jagathesan (Vivekananda College), B. Ramakrishnan (HRI, Allahabad), Sandeep E. M (KSoM)

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