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Polynomial Identities in Algebras

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Polynomial Identities in Algebras

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Preface

This volume contains the proceedings of the INDAM workshop on "Polynomial Identities in Algebras" held in Roma from September 16 to September 20, 2019. The purpose of the workshop was to present the current state of the art in the theory of PI-algebras.

The theory started with the discovery of special identities and with various structure theorems for primitive or prime rings satisfying a PI. Then, some deep results analyzing mainly the nil part of an algebra were proved leading to the theorem of Razmyslov on the nilpotency of the radical of a finitely generated PI-algebra over a field. A further major step was made by Kemer who developed a theory of varieties, leading to the solution of the Specht problem stating the finite generation of T-ideals in characteristic zero. The theory of Kemer introduced superalgebras and their superidentities as an essential tool. It turns out that the Grassmann algebra plays an important role and a basic result of Kemer states that a PI-algebra is PI equivalent to the Grassmann envelope of a finite-dimensional superalgebra.

Based on these grounds, the theory developed via two different methods: a geometric approach strongly related to invariants of matrices leading to the theory of trace identities and a combinatorial approach based on the representation theory of the symmetric group leading to the distinction of T-ideals through the analysis of some growth functions attached to them.

The workshop, inspired by the review of the classical results made in the last few years, revealed new perspectives and connections to other branches of mathematics suitable for the development of the theory.

The meeting brought together experts from different areas related to the theory of polynomial identities and focused on the computational and combinatorial aspects of the theory, its connection with invariant theory, representation theory, growth problems, and many other topics.

It was attended by experts from several countries, including Belgium, Brazil, Bulgaria, Canada, Israel, Poland, Russia, Ukraine, and the USA. The workshop featured 1-h lectures by E. Aljadeff, Y. Bahturin, A. Berele, V. Drensky, A. Giambruno, A. Kanel-Belov, P. Koshlukov, V. Petrogradsky, C. Polcino Milies, C.

Procesi, L. H. Rowen, and M. Zaicev and several other invited talks of shorter length.

The workshop was also an occasion for celebrating Antonio Giambruno's 70th birthday and his contribution to the theory of polynomial identities.

The papers of most of the principal speakers and of some of the invited speakers are included in the present volume. The contents span a broad range of themes in current active research areas.

The editors thank the Istituto Nazionale di Alta Matematica "Francesco Severi" for providing funding and logistical support for the workshop. They also wish to express their appreciation to the institutions that contributed financial support: Università della Basilicata, Università di Palermo, and Università di Roma "La Sapienza."

Potenza, Italy Onofrio Mario Di Vincenzo

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