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Mathematics and Statistics for Science

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Preface

This book is aimed at university students in the sciences who are not planning on continuing with a degree in mathematics, statistics, physics, or computer science, but who nevertheless need to learn some more mathematics and statistics for their science degrees. So, for example, if you're studying a discipline such as chemistry, molecular biology, biochemistry, physiology, geography, psychology, geology, or ecology, you will find this book useful for learning the mathematics and statistics you will need in your scientific career.

And you will need to know some mathematics and statistics, nothing more certain. You won't need to know a lot of theory – no theorems or proofs or corollaries or lemmas or things like that will be required – and you might not need to know a lot of advanced techniques, but nowadays science has become so quantitative that mathematics and statistics simply cannot be avoided.

Although this is certainly true, mathematics and statistics are not always taught in a way that is appropriate for students who want to learn useful techniques, but don't need to know any theory. Therefore, in this book we have taken what we think is a very different approach. Firstly, we have covered only those topics that we believe to be directly useful to a practising scientist. If it's something that we have never used ourselves in practice, then we simply leave it out. For example, the theory of Gaussian elimination is something we learned in detail ourselves, but have never used in practice. Ever. Not one of us has solved a large linear system in any way other than using a piece of software. Sure, the software uses Gaussian elimination. If you write the software you'll need to know how to do that. But if you just want to use the software, knowledge of the underlying theory is a step too far. Similarly, who now draws two-dimensional surfaces by hand? *We* don't. In fact, we probably can't, not any more; it's been too long since we learned how to do it, but never did it again. But then, why would we want to, when a computer can do that job so much more efficiently?

Secondly, each topic is approached in a way that focuses on how that topic is applied, how it can be used in practice. We don't prove theorems, or give results and propositions; we give no lemmas. Many times we simply state a result, without proving that it's actually correct. But that again is how scientists operate. They don't need to know the detailed theoretical background of why it works. They just need to know that it *does* work, and how to use it.

Of course, there are dangers in an approach like this. It's not uncommon for scientists to study complex and difficult problems, for which basic mathematical and statistical techniques are insufficient. Sometimes, application of the basic methods can lead to errors and problems; often, an understanding of *why* the basic result is true is needed. However, these cases are the exception rather than the rule. When scientists meet problems of this nature they can learn the theory then, should they wish to do so. To present the theory too soon serves only to hinder rather than to help.

There is, unfortunately, a lot that a book of this type simply cannot cover. After all, we don't want to end up with a book so large that it could not possibly be carried around. So we assume that you're already familiar with the basic operations of arithmetic, algebra and trigonometry. We assume that you can add fractions, take square roots and cube roots, factor polynomials, sketch straight lines and parabolas, and so on.

If you have no idea of how to do these things, then we suggest a more foundational mathematics/statistics book would be a lot better for you. If you've already learned how to do these things, but aren't too confident about them – maybe you learned them at school a long time ago and you've forgotten – then, fortunately, there are many places on the internet that can help you.

One of our favourite places to brush up on mathematical and statistical stuff that you may have forgotten is the Khan Academy. This site covers a huge amount of foundation material. It has problems, solutions and instructional videos. It explains things clearly. We recommend it highly.

Another excellent site is mathsisfun.com. It's not as comprehensive as the Khan Academy site, but contains some lovely animated demonstrations that are definitely worth watching.

Not only does this book omit a lot of foundational material, it also omits some more advanced topics of great scientific importance. In particular, although we consider differential equations in Chapters 25 and 26, the presentation there is only brief and doesn't cover such important things as higher-order ordinary differential equations, systems of differential equations, or partial differential equations. Similarly, we omit practically all the theory of linear systems and matrices. These are all fundamentally important for all branches of science, but there simply isn't space to discuss everything in a single book. If you want to learn more about more advanced topics like this, we suggest one of the many books on Engineering Mathematics, such as the book by Kreyszig or the book by Greenberg. There are many other possibilities, all quite similar.

How could this book be used? Well, we authors have used it to teach a course called "Mathematics for Science" at the University of Auckland, New Zealand. This course is designed for science majors (particularly chemistry and biology majors) who don't need to do more advanced mathematics courses. We have also used the statistics parts of this text to teach a statistics course in Auckland with the similar goal of teaching science majors rather than statistics or mathematics majors. This book could also be used as a supplementary text for providing more scientific context and explanation that is typically present in an undergraduate mathematics textbook. However, it would not be all that suitable as a stand-alone text to teach a course for continuing mathematics students, as the necessary theoretical development is mostly absent.

We authors are a bit of a mixed bag. One of us (James Sneyd) is an applied mathematician who specialises in the application of mathematics to cell physiology; one of us (Rachel Fewster) is a statistician who specialises in statistical ecology; one of us (Duncan McGillivray) is a chemist who specialises in the study of biological membranes using tools like neutron and X-ray scattering. However, despite our different backgrounds and research interests, there is one thing we all have in common. Each of us has used mathematics and statistics on a daily basis to solve scientific problems. We know what kind of methods are used. We know the kinds of things that scientists need to know. And we hope that we have managed to convey some of that knowledge to you, in such a way that this book will remain relevant throughout your scientific career.

Auckland,
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James Sneyd
Rachel Fewster
Duncan McGillivray

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