Lecture Notes in Mathematics 2285

Atsushi Inoue

Tomita's Lectures on Observable Algebras in Hilbert Space

Springer

Lecture Notes in Mathematics

Volume 2285

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ISSN 0075-8434 ISSN 1617-9692 (electronic)
Lecture Notes in Mathematics
ISBN 978-3-030-68892-9 ISBN 978-3-030-68893-6 (eBook)
https://doi.org/10.1007/978-3-030-68893-6

Mathematics Subject Classification: Primary: 46L99; Secondary: 46K10

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The registered company address is: Gewerbestrasse 11, 6330 Cham, Switzerland

Dedicated to the memory of my former supervisor late Professor Minoru Tomita

Preface

In 1967, M. Tomita presented his research on “standard forms of von Neumann algebras” at the international conference on C^*-algebras and their physical applications held at the University of Louisiana and at the fifth functional analysis symposium of Mathematical Society of Japan held in Sendai. This is an original theory that is the essence of noncommutative analysis. In 1970, M. Takesaki developed this theory and published the outcome of his work under the title “Tomita’s Theory of Modular Hilbert Algebras and Its Applications” in Springer’s Lecture Notes in Mathematics. This is known as the Tomita-Takesaki theory. I was a student of Professor Tomita from 1966 to 1971, and at that time, he did not mention to us anything about Tomita-Takesaki theory. He would come into the lecture room with a few sticks of chalks and lecture on research topics like “observable algebras”, “operators and operator algebras on Krein spaces” and “noncommutative Fourier analysis” that were elaborated after the Tomita theory. At times, he was standing in front of the blackboard thinking and writing and suddenly he erased everything; it is certain that he was testing his mathematical thoughts of that moment. Personally, I could understand almost nothing of the contents of Tomita’s lectures, so I was just keeping notes. However, I think that I naturally learned from him how to think about mathematics and how to approach mathematical problems. About 10 years later, I noticed that my research topic “unbounded operator algebras” is concerned with the theory of “observable algebras” presented in his aforementioned lectures. Thus, I started attending a lecture again for about 1 year. At that time, I got a copy of the previous lecture notes kept by my colleague H. Kurose, a student of Professor Tomita too, and read it myself in order to use in my research [16]. But, Tomita was not interested in publishing his results. So, as regards his research on “operators and operator algebras on Krein spaces”, Y. Nakagami collaborated with him and the paper “Triangular Matrix Representation for Self-Adjoint Operators in Krein Spaces” was published in Japanese J. Math. in 1988 [23]. Nakagami continued his studies as the only author, resulting in the papers [21, 22]. Another of Tomita’s students, S. Ôta, studied “Lorentz algebras on Krein spaces” [24]. The theory of observable algebras is closely related to operator algebras and its related fields. The Tomita-Takesaki theory is a special case of this theory, every observable algebra can be regarded as an operator algebra on a Pontryagin space with codimension 1 and the representation theory of locally convex $*$-algebras results in this theory. From all these, one concludes that this theory provides the mathematical techniques that establish intimate connections between the operator algebras and quantum theories. Unfortunately, Professor Tomita passed away in 2015 without publishing his work on observable algebras. Afraid that this theory would perish without being used, I decided to write these notes based on his research materials:

1. Notes of Professor Kurose and myself from Tomita's lectures.
2. Harmonic analysis on topological $*$-algebras [40].
3. Algebra of observables in Hilbert space [41].
4. Fundamental of noncommutative Fourier analysis [42].

There were many unproved parts, as well unclear parts in the preceding sources. This note is a compilation of the Tomita's observable algebras within the scope of what the author, who has been studying them for many years, could achieve.
First, I would like to thank my supervisor Professor M. Tomita for giving me many mathematical ideas and for his constant warm generous attention. Many thanks are also due to Professor Y. Nakagami for checking this manuscript in detail and for giving me much advice and comments. Furthermore, I would like to thank Professors M. Fragoulopoulou, S. Ôta and M. Uchiyama for many useful and helpful suggestions. I was able to complete this manuscript by giving lectures to Dr. H. Inoue and Dr. M. Takakura on the contents of the present work, in Fukuoka University, as well as having many discussions with them. I would like to thank H. Inoue for his careful reading of my manuscript and comments.

Fukuoka, Japan Atsushi Inoue
July 2020

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