Arak M. Mathai · Serge B. Provost · Hans J. Haubold

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ISBN 978-3-030-95863-3 ISBN 978-3-030-95864-0 (eBook)
https://doi.org/10.1007/978-3-030-95864-0

This Springer imprint is published by the registered company Springer Nature Switzerland AG
The registered company address is: Gewerbestrasse 11, 6330 Cham, Switzerland

Preface

Multivariate statistical analysis often proves to be a challenging subject for students. The difficulty arises in part from the reliance on several types of symbols such as subscripts, superscripts, bars, tildes, bold-face characters, lower- and uppercase Roman and Greek letters, and so on. However, resorting to such notations is necessary in order to refer to the various quantities involved such as scalars and matrices either in the real or complex domain. When the first author began to teach courses in advanced mathematical statistics and multivariate analysis at McGill University, Canada, and other academic institutions around the world, he was seeking means of making the study of multivariate analysis more accessible and enjoyable. He determined that the subject could be made simpler by treating mathematical and random variables alike, thus avoiding the distinct notation that is generally utilized to represent random and non-random quantities. Accordingly, all scalar variables, whether mathematical or random, are denoted by lowercase letters and all vector/matrix variables are denoted by capital letters, with vectors and matrices being identically denoted since vectors can be viewed as matrices having a single row or column. As well, variables belonging to the complex domain are readily identified as such by placing a tilde over the corresponding lowercase and capital letters. Moreover, he noticed that numerous formulas expressed in terms of summations, subscripts, and superscripts could be more efficiently represented by appealing to matrix methods. He further observed that the study of multivariate analysis could be simplified by initially delivering a few lectures on Jacobians of matrix transformations and elementary special functions of matrix argument, and by subsequently deriving the statistical density functions as special cases of these elementary functions as is done for instance in the present book for the real and complex matrix-variate gamma and beta density functions. Basic notes in these directions were prepared and utilized by the first author for his lectures over the past decades. The second and third authors then joined him and added their contributions to flesh out this material to full-fledged book form. Many of the notable features that distinguish this monograph from other books on the subject are listed next.

Special Features

1. As the title of the book suggests, its most distinctive feature is its development of a parallel theory of multivariate analysis in the complex domain side by side with the corresponding treatment of the real cases. Various quantities involving complex random variables such as Hermitian forms are widely used in many areas of applications such as light scattering, quantum physics, and communication theory, to name a few. A wide readership is expected as, to our knowledge, this is the first book in the area that systematically combines in a single source the real results and their complex counterparts. Students will be able to better grasp the results that are holding in the complex field by relating them to those existing in the real field.

2. In order to avoid resorting to an excessive number of symbols to denote scalar, vector, and matrix variables in the real and complex domains, the following consistent notations are employed throughout the book: All real scalar variables, whether mathematical or random, are denoted by lowercase letters and all real vector/matrix variables are denoted by capital letters, a tilde being placed on the corresponding variables in the complex domain.

3. Mathematical variables and random variables are treated the same way and denoted by the same type of letters in order to avoid the double notation often utilized to represent random and mathematical variables as well as the potentially resulting confusion. If probabilities are to be attached to every value that a variable takes, then mathematical variables can be construed as degenerate random variables. This simplified notation will enable students from mathematics, physics, and other disciplines to easily understand the subject matter without being perplexed. Although statistics students may initially find this notation somewhat unsettling, the adjustment ought to prove rapid.

4. Matrix methods are utilized throughout the book so as to limit the number of summations, subscripts, superscripts, and so on. This makes the representations of the various results simpler and elegant.

5. A connection is established between statistical distribution theory of scalar, vector, and matrix variables in the real and complex domains and fractional calculus. This should foster further growth in both of these fields, which may borrow results and techniques from each other.

6. Connections of concepts encountered in multivariate analysis to concepts occurring in geometrical probabilities are pointed out so that each area can be enriched by further work in the other one. Geometrical probability problems of random lengths, random areas, and random volumes in the complex domain may not have been developed yet. They may now be tackled by making use of the results presented in this book.

7. Classroom lecture style is employed as this book's writing style so that the reader has the impression of listening to a lecture upon reading the material.

8. The central concepts and major results are followed by illustrative worked examples so that students may easily comprehend the meaning and significance of the stated results. Additional problems are provided as exercises for the students to work out so that the remaining questions they still may have can be clarified.

9. Throughout the book, the majority of the derivations of known or original results are innovative and rather straightforward as they rest on simple applications of results from matrix algebra, vector/matrix derivatives, and elementary special functions.

10. Useful results on vector/matrix differential operators are included in the mathematical preliminaries for the real case, and the corresponding operators in the complex domain are developed in Chap. 3. They are utilized to derive maximum likelihood estimators of vector/matrix parameters in the real and complex domains in a more straightforward manner than is otherwise the case with the usual lengthy procedures. The vector/matrix differential operators in the complex domain may actually be new whereas their counterparts, the real, case may be found in Mathai (1997) [see Chapter 1, reference list].

11. The simplified and consistent notation of $\mathrm{d}X$ is used to denote the wedge product of the differentials of all functionally independent real scalar variables in X, whether X is a scalar, a vector, or a square or rectangular matrix, with $\mathrm{d}\tilde{X}$ being utilized for the corresponding wedge product of differentials in the complex domain.

12. Equation numbering is done sequentially chapter/section-wise; for example, (3.5.4) indicates the fourth equation appearing in Sect. 5 of Chap. 3. To make the numbering scheme more concise and descriptive, the section titles, lemmas, theorems, exercises, and equations pertaining to the complex domain will be identified by appending the letter 'a' to the respective section numbers such as (3.5a.4). The notation (i), (ii), . . . , is employed for neighboring equation numbers related to a given derivation.

13. References to the previous materials or equation numbers as well as references to subsequent results appearing in the book are kept a minimum. In order to enhance readability, the main notations utilized in each chapter are repeated at the beginning of each one of them. As well, the reader may notice certain redundancies in the statements. These are intentional and meant to make the material easier to follow.

14. Due to the presence of numerous parameters, students generally find the subject of factor analysis quite difficult to grasp and apply effectively. Their understanding of the topic should be significantly enhanced by the explicit derivations that are provided, which incidentally are believed to be original.

15. Only the basic material in each topic is covered. The subject matter is clearly discussed and several worked examples are provided so that the students can acquire a clear understanding of this primary material. Only the materials used in each chapter are given as reference—mostly the authors' own works. Additional reading materials are listed at the very end of the book. After acquainting themselves with the introductory material presented in each chapter, the readers ought to be capable of mastering more advanced related topics on their own.

Multivariate analysis encompasses a vast array of topics. Even if the very primary materials pertaining to most of these topics were included in a basic book such as the present one, the length of the resulting monograph would be excessive. Hence, certain topics had to be chosen in order to produce a manuscript of a manageable size. The selection of the topics to be included or excluded is authors' own choice, and it is by no means claimed that those included in the book are the most important ones or that those being omitted are not relevant. Certain pertinent topics, such as confidence regions, multiple confidence intervals, multivariate scaling, tests based on arbitrary statistics, and logistic and ridge regressions, are omitted so as to limit the size of the book. For instance, only some likelihood ratio statistics or λ-criteria based tests on normal populations are treated in Chap. 6 on tests of hypotheses, whereas the authors could have discussed various tests of hypotheses on parameters associated with the exponential, multinomial, or other populations, as they also have worked on such problems. As well, since results related to elliptically contoured distributions including the spherically symmetric case might be of somewhat limited interest, this topic is not pursued further subsequently to its introduction in Chap. 3. Nevertheless, standard applications such as principal component analysis, canonical correlation analysis, factor analysis, classification problems, multivariate analysis of variance, profile analysis, growth curves, cluster analysis, and correspondence analysis are properly covered.

Tables of percentage points are provided for the normal, chisquare, Student-t, and F distributions as well as for the null distributions of the statistics for testing the independence and for testing the equality of the diagonal elements given that the population covariance matrix is diagonal, as they are frequently required in applied areas. Numerical tables for other relevant tests encountered in multivariate analysis are readily available in the literature.

This work may be used as a reference book or as a textbook for a full course on multivariate analysis. Potential readership includes mathematicians, statisticians, physicists, engineers, as well as researchers and graduate students in related fields. Chapters 1–8 or sections thereof could be covered in a one- to two-semester course on mathematical statistics or multivariate analysis, while a full course on applied multivariate analysis might focus on Chaps. 9–15. Readers with little interest in complex analysis may omit the sections whose numbers are followed by an ‘a’ without any loss of continuity. With this book and its numerous new derivations, those who are already familiar with multivariate analysis in the real domain will have an opportunity to further their knowledge of the subject and to delve into the complex counterparts of the results.

The authors wish to thank the following former students of the Centre for Mathematical and Statistical Sciences, India, for making use of a preliminary draft of portions of the book for their courses and communicating their comments: Dr. T. Princy, Cochin University of Science and Technology, Kochi, Kerala, India; Dr. Nicy Sebastian, St. Thomas College, Calicut University, Thrissur, Kerala, India; and Dr. Dilip Kumar, Kerala University, Trivandrum, India. The authors also wish to express their thanks to Dr. C. Satheesh Kumar, Professor of Statistics, University of Kerala, and Dr. Joby K. Jose, Professor of Statistics, Kannur University, for their pertinent comments on the second drafts of the chapters. The authors have no conflict of interest to declare. The second author would like to acknowledge the financial support of the Natural Sciences and Engineering Research Council of Canada.

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July 1, 2022

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