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Melvyn B. Nathanson *Editor*

Combinatorial and Additive Number Theory V

CANT, New York, USA, 2021

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Melvyn B. Nathanson
Department of Mathematics
Lehman College
Bronx, NY, USA

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Preface: Mathematics in the Time of COVID

Workshops on Combinatorial and Additive Number Theory (CANT) have been organized at the CUNY Graduate Center in New York every year since 2003. We would usually meet for 4 days in May, in the week immediately preceding Memorial Day. These in-person meetings had become a fixed point in the number theory calendar. CANT 2020 and CANT 2021 were different. In 2020, because of the COVID pandemic, it was impossible to meet in person at the CUNY Graduate Center, and CANT 2020 was held on Zoom. In 2021, the COVID pandemic still had not abated, and CANT 2021, the 19th annual meeting, again took place on Zoom. A mega-meeting, with over 100 talks, it ran for 5 days, from Monday, May 24, to Friday, May 28, 2021, and was a great success. Zoom attendance was high and the videos of the talks, uploaded to YouTube, continue to attract viewers.

In the coming years, historians and sociologists of science will investigate how COVID affected research in mathematics. How much does mathematics depend on international travel and face-to-face interactions? How essential are the large grants that pay for them? The pandemic affected everything, but it is likely that the future will show that research in mathematics and, probably, in most other sciences did not decline under COVID, and, in fact, benefitted in many ways.

There are five transformative inventions (all Internet-enabled) that saved science. All are recent inventions, though only one was created during and, in part, because of the COVID pandemic.

The first, which needs no introduction, is Zoom. It made online conferencing easy, efficient, and cheap. Face-to-face discussions on Zoom may convey less substantive and emotional information than in-person meetings, but the difference is probably slight.

Wikipedia and arXiv.org are online creations of enormous value to the mathematics research community. The arXiv provides free worldwide access to a vast library of preprints in all fields of mathematics and is a viable alternate to the admittedly pleasurable activity of browsing through the new journals shelf in a library. The website arXiv was created as a preprint server for articles in physics but has grown to include mathematics and some other sciences.

The amazingly broad range of mathematics articles on Wikipedia provides an effective alternative to meeting someone in the department lounge to satisfy your curiosity and learn about topics in mathematics that might be far from your areas of specialization. Wikipedia articles vary in quality and reliability, but the average article is informative and contains useful references.

The fourth factor is YouTube.com. A treasure trove of videos of mathematics lectures by distinguished researchers has been uploaded to YouTube and can be watched by anyone anywhere for free. This is an amazing resource.

The fifth and final Internet invention that importantly contributes to science is the website researchseminars.org. It was created as mathseminars.org by a group of mathematicians at MIT and launched on April 10, 2020. A few months later it was renamed researchseminars.org and extended to include other sciences. The website provides a master list of seminars and conferences (with direct videoconference links) taking place at universities and institutes around the world. Now, sitting at home with a computer and the internet, one can participate in an unimaginably broad and rich array of research seminars.

The access to information provided by Zoom, Wikipedia, arXiv, YouTube, and researchseminars.org would have been unimaginable a few years ago. COVID forced the international research community to adopt these technologies much faster and more intensely than would otherwise have happened. The cataclysmic change in the scientific environment caused by the pandemic created rapid evolutionary adaptation. In this sense, COVID benefitted science.

I hope that the online CANT conferences, with lectures listed on researchseminars.org and presented on Zoom, and with videos of the talks available on YouTube.com, contribute to the international mathematics research enterprise.

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Previous volumes in this series are [1], [2], [3], and [4].

Short Hills, NY, USA
June 2022

Melvyn B. Nathanson

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