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The European Consortium for Mathematics in Industry

Michael Günther
Wil Schilders *Editors*

Novel Mathematics Inspired by Industrial Challenges

EUROPEAN CONSORTIUM FOR MATHEMATICS IN INDUSTRY

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To our loved ones

Preface

Mathematics is essential for innovations in industry and science. In many countries, books with success stories of mathematics have been published [1], reports by independent accountants have shown the high economic value of mathematics[2,3,4], and the Study Groups of Mathematics with Industry are spread all over the world [5]. Conferences like the biennial ECMI conference, the biennial SIAM CSE conference and the ICIAM conference organized every 4 years contain a large variety of applications of mathematics to challenges from industry and other sciences.

Despite all of these success stories, it is still claimed that these successes build on existing mathematics, that no new mathematics is generated and that mathematics for industry is not challenging at all. Some even feel that new mathematics is only created by brilliant theoretical mathematicians. A prominent and well-known example is formed by the number-theoretic results obtained by the famous mathematician Hardy in the 1940s, that are now the basis of all encryption algorithms used for financial transactions. Hardy himself would never have dreamed about this, in fact he would have considered it a nightmare that his methods are used for something practical.

With this book, we wish to demonstrate that mathematics for industry is challenging and extremely rewarding, leading to new mathematical methods and sometimes even to entirely new fields within mathematics. A nice example from our own experience is the solution of indefinite linear systems which originated from working on electronic circuit simulation. Solving the large linear systems associated with electronic circuits often led to problems with pivoting. Then the idea came up to use the fact that there are two variables in the problem: currents and voltages. This inspired us to re-order the matrix in terms of 2x2 blocks, coupling voltages to cur-

[1] https://www.eu-maths-in.eu/wp-content/uploads/2017/01/2011_FLMI-EU_IndustrialMaths-SuccessStories.pdf

[2] https://www.platformwiskunde.nl/wp-content/uploads/2016/10/Deloitte-rapport-20140115-Mathematical-sciences.pdf

[3] https://www.eu-maths-in.eu/wp-content/uploads/2016/02/2015-France-SocioEconomic_Impact_of_Mathematics.pdf

[4] http://www.eu-maths-in.eu/wp-content/uploads/2019/04/MathematicsImpactStudy_Spain.pdf

[5] https://ecmiindmath.org/study-groups/

rents. The success was immediate, no pivoting was required anymore. The method was generalised, so that it is applicable to all kinds of indefinite systems, and not only those coming from electronic circuits. It led to very nice new research results, and new methods for indefinite linear systems [6,7]. Another example is the development of methods within the field of model order reduction. This field has benefited much from demands of and developments in the electronics industry. Methods like PRIMA [8] and SPRIM [9] originated here, as did many other developments, but all methods are generally applicable. In recent years we observe a much wider variety of applications of model order reduction.

This book presents methods that fall into the category sketched in the foregoing paragraph. The starting point is always an industrial challenge. The chapters describe how the authors addressed the challenge and developed new methods that were initially specific for the application, but later formulated for general application. The book is split into two parts, one on engineering applications and one on stochastics and finance.

All chapters contained in this book clearly show that industrial challenges do lead to the development of new mathematical methods, or even completely new fields of mathematics, needed to address these challenges. The starting point maybe an application of existing mathematical methods, but when it is found that more is needed, or different methods, then the interaction between application and mathematics starts. Mathematicians can then on the one hand develop new mathematical techniques, on the other hand solve the challenges. This is extremely rewarding, it often leads to nice journal papers on the theoretical results, which subsequently are the starting point of a lot of further research inside the mathematics area. It also leads to papers in applied journals.

Concluding, we may say that "mathematics for industry" or, even broader, "applied mathematics", is much more than just applying existing mathematical methods to industrial problems. In many cases, the application of existing methods does not lead to the desired solution, and hence adaptations of existing methods or even entirely new methods need to be developed in order to effectively address the industrial challenge. In some cases, this has led to entirely new fields within mathematics. The interplay between mathematics and industry is, hence, beneficial for both. Industry benefits by having their problems solved and mathematics benefits because new methods are developed that are versatile in nature.

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[6] H.S. Dollar, N.I.M. Gould, W.H.A. Schilders, A.J. Wathen: On iterative methods and implicit-factorization preconditioners for regularized saddle-point systems, SIAM J. Matr. Anal. Appl. (27) 170–189 (2006)

[7] W.H.A. Schilders: Solution of indefinite linear systems using an LQ decomposition for the linear constraints, Linear Algebra and its Applications 431:30-4 381–395 (2009)

[8] A. Odabasioglu, M. Celik and L.T. Pileggi: PRIMA: passive reduced-order interconnect macromodeling algorithm, IEEE Trans. Comp. Aid. Dsg. Int. Circ. Syst. 17:8 645–654 (1998)

[9] R.W. Freund: Structure-Preserving Model Order Reduction of RCL Circuit Equations, in: W.H.A. Schilders, H.A. van der Vorst and J. Rommes (eds): Model Order Reduction: Theory, Research Aspects and Applications, Springer Verlag, Heidelberg, 51–75 (2008)

The book contains two parts on applications in *Computational Science and Engineering* and *Data Analysis and Finance*. It should be remarked that all authors have been asked to start and end their chapter with a brief description of why their chapter fits into this volume: explaining which industrial challenges have been instrumental for their inspiration, and which methods have been developed as a result.

Wuppertal and Eindhoven, *Michael Günther*
Spring 2021 *Wil Schilders*

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