

Outstanding Contributions to Logic 24

Alex Citkin

Ioannis M. Vandoulakis *Editors*

V. A. Yankov on Non-Classical Logics, History and Philosophy of Mathematics

 Springer

Outstanding Contributions to Logic

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Editor-in-Chief

Sven Ove Hansson, Division of Philosophy, KTH Royal Institute of Technology,
Stockholm, Sweden

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Editors

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Editors

Alex Citkin
Metropolitan Telecommunications
New York, NY, USA

Ioannis M. Vandoulakis
Hellenic Open University
Athens, Greece

FernUniversität in Hagen
Hagen, Germany

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Preface

This volume is dedicated to Vadim Yankov (Jankov¹), the Russian logician, historian and philosopher of mathematics and political activist who was prosecuted in the former USSR.

In 1964, he defended his dissertation *Finite implicative structures and realizability of formulas of propositional logic* under the supervision of A. A. Markov. In the 1960s, Yankov published nine papers dedicated to non-classical propositional logics, predominantly to intermediate logics. Even today, these publications—more than fifty years later—still hold their place among the most quotable papers in logic. The reason for this is very simple: not only Yankov obtained significant results in propositional logic, but he also developed a machinery that has been successfully used to obtain new results up until our days.

Yankov studied the class of all intermediate logics, as well as some particular intermediate logic. He proved that the class of all intermediate logic ExtInt is not denumerable, and that there are intermediate logics lacking the finite model property, and he had exhibited such a logic. In addition, he proved that ExtInt contains infinite strongly ascending, strongly descending and independent (relative to set inclusion) subclasses of logics, each of which is defined by a formula on just two variables. Thus, it became apparent that ExtInt as a lattice has a quite a complex structure.

In 1953, G. Rose gave a negative answer to a hypothesis that the logic of realizability, introduced by S. Kleene in an attempt to give precise intuitionistic semantics to Int , does not coincide with Int . In 1963, Yankov constructed the infinite series of realizable formulas not belonging to Int .

In his 1968 paper, Yankov studied the logic of the weak law of excluded middle, and the logic defined relative to Int by a single axiom $\neg p \vee \neg\neg p$. Nowadays, this logic is often referred to as a Yankov (or Jankov) logic. In particular, Yankov has discovered that this logic has a very special place in ExtInt : it is the largest logic,

¹ In Russian, the last name is ЯНКОВ. In the translations of papers of the 1960s by the American Mathematical Society, the last name was transliterated as “Jankov,” while in the later translations, the last name is transliterated as “Yankov,” which perhaps is more correct. In this volume, the reader will see both spellings.

a positive fragment of which coincides with the positive fragment of *Int*, while all extensions of the Yankov logic have distinct positive fragments.

In his seminal 1969 paper, Yankov described in detail the machinery mentioned above. The reader can find more on Yankov's achievements in intermediate logics in the exposition included in this volume.

However, not only Yankov's results in studying intermediate logics are important. His papers instigate the transition from matrix to algebraic semantics. Already in his 1963 papers, he started to use what is now known as Heyting or pseudo-Boolean algebras. At the same time, H. Rasiowa and R. Sikorski's book, *The Mathematics of Methamematics*, was published, in which the pseudo-Boolean algebras were studied. Yankov made the Russian translation of this book (published in 1972), and it greatly influenced the researchers in the former Soviet Union. Besides, Yankov was one of the pioneers who studied not only intermediate logics—extensions of *Int*, but also extensions of positive and minimal logics and their fragments. It would not be an overstatement to say that Yankov is one of the most influential logicians of his time.

At the end of the 1960s and in the 1970s, Yankov got more involved in the political activities. In 1968, he joined other prominent mathematicians and co-signed the famous letter of the 99 Soviet mathematicians addressed to the Ministry of Health and the General Procurator of Moscow asking for the release of imprisoned Esenin-Vol'pin. As a consequence, Yankov lost his job at the Moscow Institute of Physics and Technology (MIPT), and most of the mathematicians who signed this letter faced severe troubles.

Since 1972, he started to publish abroad, for instance, in the dissident journal *Kontinent*, founded in 1974 by writer Vladimir Maximov that was printed in Paris and focused on the politics of the Soviet Union. In issue 18, he published the article "On the possible meaning of the Russian democratic movement." In 1981–1982 he wrote a "Letter to Russian workers on the Polish events," on the history and goals of the "Solidarity" trade union. Following these events, he was arrested in August 1982, and on January 21, 1983, the Moscow City Court sentenced him to four years in prison and three years in exile for anti-Soviet agitation and propaganda. He served his term in the Gulag labor camp, called "Dubravny Camp" in Mordovia, near Moscow, and exile in Buryatia in south-central Siberia. He was released in January 1987 and rehabilitated in 1991.

Despite his hard life in the Camp and the exile, Yankov started to study philosophy and the classic Greek language. The second editor was impressed when he visited him at home in Dolgoprudnyj, near Moscow, in 1990, and Yankov started to analyze the syntax of a passage from Plato's *Parmenides* in classic Greek. When he asked him where he studied classical Greek so competently, he was stunned Yankov's unexpected answer: "In prison"!

Thus, Yankov's philosophical concerns were shaped while he was imprisoned. His first, possibly philosophical publication was printed abroad in issue 43 (1985) of the journal *Kontinent*, entitled "Ethical-philosophical treatise," where he outlines his philosophical conception of existential history. A publication on the same theme in Russia was made possible only ten years later, in the journal *Voprosy Filosofii* (1998, 6).

After Yankov's acquaintance with the second editor's Ph.D. Thesis, he agreed to become a member of the Committee of Reviewers and then started to examine the history of Greek mathematics systematically but from a specific logical point of view. He was primarily concerned about the ontological aspects of Greek mathematical theories and the relevant ontological theories in pre-Socratic philosophy. He stated a hypothesis on the rise of mathematical proof in ancient Greece, which integrated into the broader context of his inquiry of the pre-Socratic philosophy.

This volume is a minimal appreciation to a mathematician and scholar who deserves our respect and admiration.

New York, USA
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Alex Citkin
Ioannis M. Vandoulakis

Contents

1	Short Autobiography	1
	Vadim A. Yankov	
Part I Non-Classical Logics		
2	V. Yankov’s Contributions to Propositional Logic	7
	Alex Citkin	
3	Dialogues and Proofs; Yankov’s Contribution to Proof Theory	53
	Andrzej Indrzejczak	
4	Jankov Formulas and Axiomatization Techniques for Intermediate Logics	71
	Guram Bezhanishvili and Nick Bezhanishvili	
5	Yankov Characteristic Formulas (An Algebraic Account)	125
	Alex Citkin	
6	The Invariance Modality	165
	Silvio Ghilardi	
7	The Lattice NExtS41 as Composed of Replicas of NExtInt, and Beyond	177
	Alexei Muravitsky	
8	An Application of the Yankov Characteristic Formulas	209
	Valery Plisko	
9	A Note on Disjunction and Existence Properties in Predicate Extensions of Intuitionistic Logic—An Application of Jankov Formulas to Predicate Logics	221
	Nobu-Yuki Suzuki	

Part II History and Philosophy of Mathematics

10 On V. A. Yankov’s Contribution to the History of Foundations of Mathematics 247
Ioannis M. Vandoulakis

11 On V. A. Yankov’s Existential Interpretation of the Early Greek Philosophy. The Case of Heraclitus 271
Tatiana Yu. Denisova

12 On V. A. Yankov’s Hypothesis of the Rise of Greek Mathematics ... 295
Ioannis M. Vandoulakis

Index 311