CIMAT Lectures in Mathematical Sciences

Gerardo Arizmendi Echegaray
Luis Hernández-Lamoneda
Rafael Herrera Guzmán
Editors

Recent Advances in Alexandrov Geometry

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Preface

The Eleventh Mini-Meeting on Differential Geometry was held from December 10th to 12th, 2018, at the Center for Research in Mathematics (CIMAT), Guanajuato, México.

The invited speakers included Peter Petersen (Distinguished Visiting Professor for the Mexican Academy of Sciences and the USA-Mexico Foundation for Science), Fernando Galaz-García, Jesús Angel Núñez-Zimbrón, Gabriel Ruiz, Gregor Weingart, Miguel Angel García-Ariza, Rosemberg Toalá-Enríquez and Armando Cabrera-Pacheco. Peter Petersen and Fernando Galaz-García delivered two advanced mini-courses of three lectures each on Alexandrov Geometry, while the rest of the participants delivered research talks on diverse topics.

This volume is devoted to various aspects of Alexandrov Geometry for those wishing to get a detailed picture of the advances in the field. It contains enhanced versions of the lecture notes of the two mini-courses plus those of one research talk:

- Peter Petersen's notes aim at presenting various rigidity results about Alexandrov spaces (work with Karsten Grove) in a way that facilitates the understanding by a larger audience of geometers of some of the current research in the subject. They contain a brief overview of the fundamental aspects of the theory of Alexandrov spaces with lower curvature bounds, as well as the aforementioned rigidity results with complete proofs.
- The lecture notes of Fernando Galaz-García's mini-course were completed in collaboration with Jesús Nuñez-Zimbrón. They present an up-to-date and panoramic view of the topology and geometry of three-dimensional Alexandrov spaces, including the classification of positively and non-negatively curved spaces and the geometrization theorem. They also present Lie group actions and their topological and equivariant classifications as well as a brief account of results on collapsing Alexandrov spaces.
- Jesús Nuñez-Zimbrón's notes survey two recent developments in the understanding of the topological and geometric rigidity of singular spaces with curvature bounded below.

We thank all the participants for making the meeting a success. We also thank CIMAT's staff for their help in the organization and smooth running of the event. This meeting was the eleventh edition of an annual event intended for researchers and graduate students, with the dual aim of combining a winter school and a research workshop. The meeting was supported by the Mexican Academy of Sciences (AMC), the USA-Mexico Foundation for Science (FUMEC), the Universidad de las Américas Puebla (UDLAP), the Mexican Science and Technology Research Council (CONACyT) and the Center for Research in Mathematics (CIMAT). The organizers were Gerardo Arizmendi (UDLAP, México), Luis Hernandez-Lamoneda (CIMAT, México) and Rafael Herrera (CIMAT, México).

Cholula, México — Gerardo Arizmendi Echegaray
Guanajuato, México — Luis Hernández-Lamoneda
Guanajuato, México — Rafael Herrera Guzmán

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