Alfio Quarteroni

Modeling Reality with Mathematics

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to Lara, Luca, and Bianca Sofia,
my little models

Preface

Following the dramatic spread of the COVID-19 pandemic at the start of 2020, mathematics has never been so much at the centre of everybody's attention.

Expressions like *exponential*, *logistic function*, *extrema* and *inflection*, which until that moment we thought were confined to classrooms and university halls, all of a sudden have entered the political debate. This was all the more true in the initial stages of the epidemic, which were characterised by a major uncertainty regarding the contagion's evolution.

Yet even a casual observer must have noticed that mathematics is present in an increasingly pervasive way in our daily lives. There is chatter in the media about intangible algorithms for finding a soulmate, concocting the perfect diet or taking any decision whatsoever. The news report of billions of euros going up in flames on stock exchanges around the world due to an algorithm that went amok. They boast how *big data* (which everybody talks about, although very few truly know about) is essential for economic and technological advancement.

In this book, as you may have gathered from the title, I intend to present a version of mathematics that is less hostile and obscure. Or rather, I will present an area, called mathematical modelling, that over the last few decades has taken the front row. Whether we are aware of it or not (as is more often the case), we all regularly benefit from mathematical models and algorithms, and without them our lives would be very different. Here are some examples. Without mathematical models we would not know what the weather will look like tomorrow. We would not be able to share photos and videos on our phones, nor browse the web as fast as we do. We could not blindly rely on sat-navs to find our way through cities we have never been to before. Our cars would not be so silent, comfortable and efficient. We could not use CAT scans to take a look inside our bodies, nor would our favourite football team have a legion of *match analysts* studying strategies to boost competitiveness using the omnipresent *big data*. The list of similar stunning achievements could go on and on.

Mathematics is—even if you do not know, it is easy to guess so—an abstract science. And that is precisely one of its secret weapons. Abstraction allows us to study problems in their full generality, and helps us understand their key and innermost features. Abstraction tickles our imagination and imagination nurtures creativity, which in turn allows us to discover the best path towards solving our problems. We rely on abstraction to formulate far-fetching conjectures, which sometimes are so intricate that they withstand our attempts to solve (or disprove) them for centuries. Fermat's last theorem, for instance, was stated in 1637 and only proved by the British mathematician Andrew Wiles in 1994. The Riemann hypothesis was first published in 1859 and is still unsolved.

Mathematical models are characterised by being concrete, because they need to be both useful and of broad interest. As strange as it may sound, they are born and they flourish precisely because of abstraction. To better understand this crucial fact and reveal it in all its might, in the next pages we shall attempt to clarify what a mathematical model truly is, and present some examples.

Before that, though, we should recall the pivotal role of two core players in mathematics, which all of us have met in school, namely numbers and equations. *Numbers* allow us to quantify distances, weights, time intervals and so on. *Equations* describe in a general way the relationships that govern natural processes (the movement of a glacier, the flooding caused by a river's surge, the propagation of seismic waves but also solar combustion and even how a forest grows). Equations allow us to compute the trajectories of satellites and the courses of Formula 1 racing cars; they manage industrial processes and regulate the negotiation of complex financial instruments. Because of equations we are able to produce wonderful animated films (whose characters and events, albeit amazingly realistic, are solutions to mathematical equations), we can study the best tactic for a volleyball team and we can even simulate how vital organs like the heart or the brain work. All of this is possible because equations translate into symbols and mathematical relations physical laws, biological processes and social behaviours. In this way, they allow us to construct a virtual world (the model) starting from the real world. By solving equations we can make predictions (think of weather forecasts), simulate the progression of an illness or calculate how a volcanic eruption will disperse its ashes in the atmosphere. Summing up, a mathematical model's equations are telescopes pointed into the future.

Mathematics is the oldest among the sciences (the earliest written documents were found in ancient Egypt and Mesopotamia roughly 2000 BCE), and we can bet on it being the one that will survive them all. Mathematics is the only field capable of developing autonomously, since all other sciences require the language and the tools of mathematics to express themselves. We had better acknowledge its role then. If you are willing to accompany me on this journey, I intend to help you look at mathematics through a different lens. Hopefully, I will awaken (once more?) the interest of those of you who "In school I did understand maths, but there was a teacher who made me hate it." (In everyone's life there is always a teacher who made us love or hate a subject.)

So let the adventure begin: enjoy!

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