Trends in Mathematics

Richard M. Aron
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Editors

Operator and Norm Inequalities and Related Topics

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Operator and Norm Inequalities and Related Topics

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Preface

Inequalities play a central role in mathematics with various applications in other disciplines. The main goal of this contributed volume is to present several important matrix, operator, and norm inequalities in a systematic and self-contained fashion.

The volume includes contributions by a number of the world's leading specialists in functional analysis and operator theory. It contains the latest developments of significant mathematical inequalities in numerous fields in the last decades that are of interest to a wide audience of pure and applied mathematicians.

This book consists of 5 parts and includes a total of 23 chapters. The chapters are written in a reader-friendly style and can be read independently. Each chapter contains a rich bibliography.

Part I: Matrix and Operator Inequalities

Whenever we see an inequality concerning real or complex numbers, an interesting question is to ask ourselves whether it is true for matrices or bounded linear operators on a Hilbert space. This is based on the fact that the real linear space of self-adjoint operators (Hermitian matrices) can be regarded as a generalization of the real line. One of the most significant notions in this part is the concept of operator monotone function, which was first studied by C. Löwner [Math. Z. 38 (1934), 177–216], and its connection with operator means was introduced by F. Kubo and T. Ando [Math. Ann. 246 (1980), no. 3, 205–224]; [cf. Simon, Barry. Loewner's theorem on monotone matrix functions. Grundlehren der mathematischen Wissenschaften, 354. Springer, Cham, 2019].

Chapter "Log-majorization Type Inequalities" is devoted to studying the link between majorization theory and several matrix inequalities such as Araki's log majorization, the Löwner–Heinz, the Furuta, the Golden-Thompson, the von Neumann trace, and their extensions.

In Chapter "Ando-Hiai Inequality: Extensions and Applications", extensions and applications of the Ando-Hiai inequality are investigated and the Furuta, the Bebiano–Lemos–Providência, and the grand Furuta inequalities are explored.

Chapter "Relative Operator Entropy" demonstrates the relative operator entropy that is the tangent vector of the geodesic in the manifold of positive invertible operators. Tsallis relative entropy is studied as the secant of a path of geometric matrix means.

Chapter "Matrix Inequalities and Characterizations of Operator Monotone Functions" includes various characterizations of operator monotone functions using matrix inequalities involving matrix means. A trace monotonicity inequality and the Powers-Størmer inequality are used to characterize operator monotone functions.

Chapter "Perspectives, Means and Their Inequalities" focuses on the operator perspective and its extensions including operator means and the Pusz–Woronowicz functional calculus.

Chapter "Cauchy–Schwarz Operator and Norm Inequalities for Inner Product Type Transformers in Norm Ideals of Compact Operators, with Applications" provides an overview of operator and norm inequalities of Cauchy–Schwarz type for strongly square integrable operator families and symmetrically norming functions. Some applications to the Aczél–Bellman, Grüss–Landau, arithmetic–geometric, Young, Heinz, Heron inequalities are presented.

Chapter "Norm Estimations for the Moore-Penrose Inverse of the Weak Perturbation of Hilbert C^*-module Operators" defines multiplicative perturbations and studies representations and norm estimations for the Moore-Penrose inverse associated with the multiplicative perturbation.

Part II: Orthogonality and Inequalities

There are several ways to extend the notion of orthogonality from inner product spaces to the framework of normed spaces. The most developed one is the Birkhoff–James orthogonality. It was introduced by Birkhoff [Duke Math. J. 1 (1935), 169–172] and extensively studied by R.C. James [Duke Math. J. 12 (1945), 291–302]; cf. C. Alsina, J. Sikorska, M.S. Tomás [Norm derivatives and characterizations of inner product spaces. World Scientific Publishing Co. Pte. Ltd., Hackensack, NJ, 2010].

Chapter "Birkhoff–James Orthogonality: Characterizations, Preservers, and Orthogonality Graphs" reviews the Birkhoff–James orthogonality starting from historical perspectives throughout the current development and presents several characterizations of Birkhoff–James orthogonality in classical Banach spaces, C^*-algebras, and Hilbert C^*-modules. In addition, some characterizations of preservers of Birkhoff–James orthogonality are given.

Chapter "Approximate Birkhoff-James Orthogonality in Normed Linear Spaces and Related Topics" is an introduction to approximate Birkhoff–James orthogonality in real normed spaces and its characterizations.

Chapter “Orthogonally Additive Operators on Vector Lattices” focuses on the vector lattice structure of different partial subclasses of the vector space of all orthogonally additive operators, certain domination problems, representation theorems, and Banach lattice structure of orthogonally additive operators.

Part III: Inequalities Related to Types of Operators

This part mainly studies inequalities concerning closed range, normal, and Toeplitz operators (cf. A. Böttcher and B. Silbermann [Analysis of Toeplitz operators. Second edition. Prepared jointly with Alexei Karlovich. Springer Monographs in Mathematics. Springer-Verlag, Berlin, 2006] and I. Gohberg, S. Goldberg, and M. A. Kaashoek [Basic classes of linear operators. Birkhäuser Verlag, Basel, 2003]).

In Chapter “Normal Operators and Their Generalizations”, some aspects of local spectral theory and Fredholm theory of certain classes of operators that generalize normal operators on Hilbert spaces are studied.

Chapter “On Wold Type Decomposition for Closed Range Operators” surveys Wold-type decomposition for closed range operators satisfying certain operator inequalities. Several results on left invertible operators close to isometries are listed and extended to the case of regular operators.

Chapter “(Asymmetric) Dual Truncated Toeplitz Operators” considers properties of asymmetric dual truncated Toeplitz operators acting between the orthogonal complements of two model spaces.

Chapter “Boundedness of Toeplitz Operators in Bergman-Type Spaces” is devoted to the open problem of characterization of the bounded Toeplitz operators T_a in Bergman spaces. Based on the structure of the Bergman spaces, a characterization of the boundedness and compactness is presented in the case of operators in spaces with weighted sup-norms.

Part IV: Inequalities in Various Banach Spaces

This part deals with miscellaneous inequalities concerning topological and geometrical properties of various Banach spaces and operator algebras (cf. W.B. Johnson and J. Lindenstrauss (ed.) [Handbook of the geometry of Banach spaces. Vol. I. North-Holland Publishing Co., Amsterdam, 2001]).

In Chapter “Disjointness Preservers and Banach-Stone Theorems”, the so-called weak and strong Banach-Stone theorems are given. In addition, it is proved that in many cases lattice isomorphisms (Kaplansky’s Theorem), ring isomorphisms (Gelfand-Kolmogorov Theorem), multiplicative isomorphisms (Milgram’s Theorem), isometries (Banach-Stone Theorem), and nonvanishing preservers are $\perp$-isomorphisms.

Chapter "The Bishop–Phelps–Bollobás Theorem: An Overview" provides a comprehensive survey of the Bishop–Phelps–Bollobás theorem from 2008 to 2021.

Chapter "A New Proof of the Power Weighted Birman–Hardy–Rellich Inequalities" introduces a new proof of the optimal version of the power-weighted Birman–Hardy–Rellich integral inequalities. Extensions to homogeneous Sobolev spaces and the vector-valued case are also discussed.

Chapter "An Excursion to Multiplications and Convolutions on Modulation Spaces" is devoted to reviewing results on boundedness for multiplications and convolutions in (quasi-)Banach modulation spaces of ultradistributions. Furthermore, the Gabor product is investigated.

Chapter "The Hardy-Littlewood Inequalities in Sequence Spaces" presents modern proofs of m-linear versions of the results of Hardy and Littlewood and the state of the art of the subject, as well as an application to the combinatorial Gale-Berlekamp switching game.

Chapter "Symmetries of C^*-algebras and Jordan Morphisms" illustrates interrelations between symmetries of various structures attached to C^*-algebras and von Neumann algebras and Jordan $*$-isomorphisms. In this direction, one-dimensional projections in a Hilbert space with transition probability, projection lattices of von Neumann algebras, and measures on state spaces endowed with the Choquet order are extensively studied.

Part V: Inequalities in Commutative and Noncommutative Probability Spaces

This part includes generalizations of Doob's maximal inequality, the Burkholder–Davis–Gundy inequality, and several inequalities related to Markov processes and noncommutative free probability (cf. P. Jorgensen and F. Tian [Non-commutative analysis. With a foreword by Wayne Polyzou. World Scientific Publishing Co. Pte. Ltd., Hackensack, NJ, 2017]).

In Chapter "Mixed Norm Martingale Hardy Spaces and Applications in Fourier Analysis", martingale Hardy spaces defined with the help of mixed $L_{\vec{p}}$ -norm are investigated. Two different generalizations of Doob's maximal inequality for mixed-norm $L_{\vec{p}}$ spaces and two versions of atomic decompositions are given. Several martingale inequalities and a generalization of the Burkholder–Davis–Gundy inequality are also presented. As an application in Fourier analysis, the boundedness of the Fejér maximal operator from $H_{\vec{p}}$ to $L_{\vec{p}}$, whenever $1/2 < \vec{p} < \infty$ is obtained.

Chapter "The First Eigenvalue for Nonlocal Operators" presents some results concerning the first eigenvalue for a nonlocal operator in convolution form with a smooth kernel and gives information on the asymptotic behavior of some natural Markov processes.

Chapter "Comparing Banach Spaces for Systems of Free Random Variables Followed by the Semicircular Law" studies certain Banach-space operators from noncommutative free probability, acting on systems of free random variables whose free distributions are followed by the semicircular law.

The editors are grateful for the hard work of numerous mathematicians who carefully reviewed the chapters and gave insightful comments to improve them.

The book can be used as an introduction to several active research areas within operator theory. It is intended for use by both researchers and graduate students in mathematics, scientific computing, physics, statistics, and engineering who have a basic grasp of the fundamentals in functional analysis and operator theory.

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