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Taka-aki Shiraishi

Multiple Comparisons for Bernoulli Data



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Preface

Multiple tests and simultaneous confidence intervals specify differences in means. Tukey (1953), Miller (1981), Hochberg and Tamhane (1987), Hsu (1996), and Shiraishi et al. (2018), (2019) are some technical books on multiple comparisons for continuous random variables. Multiple comparisons for discrete random variables have been discussed a little. The present book focuses on progressive multiple comparisons of proportions in multi-sample models with Bernoulli responses.

In Chap. 1, we give theoretical basics in one-sample and two-sample models. We state regularity conditions of exact confidence limit using F-distribution. We introduce asymptotic theory based on variance-stabilizing transformation. In Chap. 2, we give simultaneous inference for all proportions in multi-sample models by using the exact confidence limit and the asymptotic theory. In Chap. 3, we discuss all-pairwise multiple comparison tests of proportions. Closed testing procedures based on maximum absolute values of some two-sample test statistics and based on χ^2 -test statistics are introduced in multi-sample models. The results suggest that the multi-step procedures are more effective than single-step procedures and the Ryan–Einot–Gabriel–Welsch (REGW) type tests. In Chap. 4, we give multiple comparison test procedures with a control. In Chap. 5, we propose simultaneous confidence intervals for difference of proportions, odds ratio, and ratio of proportions. By the theory of these intervals, we are able to obtain all-pairwise multiple comparison tests of odds ratio. In Chaps. 6 and 7, under simple ordered restrictions of proportions, we also discuss closed testing procedures based on maximum values of two-sample one-sided test statistics and based on Bartholomew’s $\bar{\chi}^2$ -statistics. Although single-step multiple comparison procedures are utilized in general, the closed testing procedures stated in the present book are fairly more powerful than the single-step procedures. Bretz et al. (2011) discuss serial gatekeeping methods based on Bonferroni inequality.

In Chaps. 8 and 9, we propose serial gatekeeping methods based on the closed testing procedures stated in Chaps. 3, 4, 6 and 7. It is shown that the proposed serial gatekeeping methods are much superior to the serial gatekeeping methods based on Bonferroni tests.

Nagoya, Japan
March 2022

Taka-aki Shiraishi

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Contents

1	Theoretical Basics in One-Sample and Two-Sample Models	1
1.1	Exact Theory in One-Sample Model	1
1.2	Asymptotic Theory in One-Sample Model	8
1.3	Asymptotic Theory in Two-Sample Model	11
	References	17
2	Simultaneous Inference for All Proportions	19
2.1	Multi-sample Models	19
2.2	Exact Conservative Procedures	20
2.2.1	Single-Step Methods	20
2.2.2	Multi-step Methods	24
2.3	Asymptotic Theory	25
2.3.1	Single-Step Methods	25
2.3.2	Sequentially Rejective Multiple Test Procedures	29
	References	29
3	All-Pairwise Comparison Tests	31
3.1	Introduction	31
3.2	Single-Step Test Procedures	32
3.3	Closed Testing Procedures	35
	References	43
4	Multiple Comparison Tests with a Control	45
4.1	Introduction	45
4.2	The Single-Step Procedures	46
4.3	Closed Testing Procedures	48
	References	51
5	Simultaneous Confidence Intervals	53
5.1	Introduction	53
5.2	Pairwise Differences of Proportions Based on Arcsine Transformation	53
5.3	All-Pairwise Differences of Proportions	55

5.4 Odds Ratios and Relative Risks of Proportions 57

Reference 59

6 All-Pairwise Comparisons Under Simple Order Restrictions 61

6.1 Introduction 61

6.2 $\bar{\chi}^2$ -Test 62

6.3 Single-Step Procedures 65

6.4 Closed Testing Procedures 66

References 73

7 Comparisons with a Control and Successive Comparisons Under Simple Order Restrictions 75

7.1 Introduction 75

7.2 Multiple Comparison Tests with a Control 76

7.3 Successive Comparisons Between Ordered Proportions 78

References 84

8 Hybrid Serial Gatekeeping Procedures for All-Pairwise Comparisons 87

8.1 Introduction 87

8.2 Multiple Comparisons Under Unrestricted Proportions in the r -th Multi-sample Model 89

8.3 Comparisons Under Order Restricted Proportions in the r -th Multi-sample Model 92

8.4 Serial Gatekeeping Procedures 98

8.5 Application to Multivariate Multi-sample Models 102

8.6 Discussion 102

References 103

9 Hybrid Serial Gatekeeping Procedures for Multiple Comparisons with a Control 105

9.1 Introduction 105

9.2 Multiple Comparisons Under Unrestricted Proportions in the r -th Multi-sample Model 107

9.3 Comparisons Under Order Restricted Proportions in the r -th Multi-sample Model 110

9.4 Serial Gatekeeping Procedures 112

9.5 Application to Multivariate Multi-sample Models 115

References 116

Index 117