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Patrizia Donato
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Emerging Problems in the Homogenization of Partial Differential Equations

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Preface

This book contains some of the results presented at the minisymposium titled *Emerging Problems in the Homogenization of Partial Differential Equations*, held during the ICIAM2019 conference in Valencia in July 2019. The quality of the works presented, as well as the successful reception of the minisymposium among those attending its sessions, led its organizers to accept Springer's invitation to publish this volume.

In order to introduce nonspecialist readers to the subject, let us mention that the aim of the mathematical homogenization is to model microscopically heterogeneous media, providing macroscopic models that describe their effective behavior. As an example of these media, we can consider composite materials, which are characterized by the fact that they contain two or more finely mixed constituents. These materials are widely used nowadays in industries, due to their properties. Indeed, they have in general a better behavior than the average behavior of its constituents. Well-known examples are the multifilamentary superconducting composites that are used in the composition of optical fibers.

Generally speaking, in a composite the heterogeneities are small compared with its global dimension. So, two scales characterize the material: the microscopic one, describing the heterogeneities, and the macroscopic one, describing the global behavior of the composite. From the macroscopic point of view, the composite looks like a *homogeneous* material. The aim of homogenization is precisely to give the macroscopic properties of the composite by taking into account the properties of the microscopic structure. From the mathematical point of view, this leads to study the asymptotic behavior of a Partial Differential Equation (PDE) depending on a small parameter, here denoted by ε, as ε tends to zero. The parameter describes the heterogeneities, which are "small" compared with the global size of the material. The main questions arising in this asymptotic analysis are:

- Does the solution of the PDE converge to some limit function?
- If that is true, does the limit function solve some limit boundary value problem and can we describe it explicitly?

Answering these questions is the aim of the mathematical theory of homogenization. Let us point out that since the coefficients of the PDE describe the characteristic of the material at the microscale, it is not realistic to suppose that the coefficients are smooth, for instance, continuous. Consequently, in general, the suitable framework is that of weak solutions in Sobolev spaces and variational formulations. Hence, the main challenge when passing to the limit is how to deal with products of two (or more) weakly convergent functions, which, as well know, do not converge to the product of their weak limit.

Perhaps the most classic homogenization problem appears when we study thermal properties of composite materials whose constituents are periodically distributed, and we have to pass to the limit in a second-order PDE with ε-periodic rapidly oscillating coefficients describing the stationary heat diffusion. When the heterogeneities are periodically distributed, the limit problem, the so-called homogenized problem, has constant coefficients, which can be described explicitly. Somehow this means that for small values of the parameter ε, the behavior of the composite material can be properly approximated by the behavior of another simpler material (a homogeneous material). Replacing the oscillating problem by the homogenized one allows to analyze the properties of the composite material in a much less complicated way, and in particular, to make easier numerical simulations. These classic results for the periodic setting have been later generalized to random composite material (i.e., the constituents are distributed in the medium according to a certain statistical law) and even to arbitrarily heterogeneous materials (which are neither periodically nor statistically homogeneous).

Composite materials are simply a kind of a wide variety of microscopically heterogeneous media, which can be studied by means of the mathematical homogenization theory. Others appear when we consider materials with holes and/or oscillating boundaries. This is, for example, the case of reticulated structures, made of thin beams or plates periodically distributed, which are very common in engineering and architecture. The study of these materials leads to consider PDE posed in perforated domains, with holes of size $\delta(\varepsilon)$ and distributed with ε-periodicity in each axis direction (here, $\delta(\varepsilon)$ is another positive parameter smaller than ε). In this setting, additional difficulties arise, since the PDE is posed in a varying domain and then its solution belongs to a Sobolev space that also depends on the small parameter ε. Remark that these homogenization problems play a main key in the optimal design of materials, when studying the limit behavior of minimizing sequences in order to prove the existence of an optimal shape or to obtain a relaxed formulation.

Although the first applications of homogenization come from engineering, its use is increasingly frequent in other fields. This is due to the fact that the theory constitutes a powerful mathematical tool for the study of complex systems that presents several elements (characteristics, constituents, etc.) of very different scales, very heterogeneous, and homogenized models provide an effective (macroscopic) description of the system that takes into account the influence of the different scales. Consequently, nowadays, the homogenization theory is used actually almost in any discipline, that is, the traditional ones such as physics, engineering, mechanics, chemistry, and economy but also biology and medicine. Let us highlight the case of the health sciences, where the systems under study are so highly complex that it is extremely difficult to study them using models that collect all scales (e.g., the blood flow in vessels, the transfer of oxygen from the alveoli of the lungs to the blood and of carbon dioxide conversely, and the skin pores among other).

The mathematical theory of homogenization has been well founded and widely developed from different approaches in the last decades. Once wide performance methods were well established, more and more researchers have expressed a high interest in them and in their application to new challenging problems, dealing with more realistic and complex models, as well as with more difficult mathematical problems. These new challenges, in turn, are an inspiration for method's improvement and development.

The aim of the minisymposium *Emerging Problems in the Homogenization of Partial Differential Equations* was to put together renowned specialists from all over the world overcovering a wide range of emerging challenging problems in the field. We have been very fortunate that our invitations to give a talk were accepted by so many well-known mathematicians, working in so many different countries and on a large spectrum of areas and problems and using many different methods. In gratitude, we recall below the list of authors of all the communications presented (we highlight the speakers in bold):

– Marc Briane (INSA de Rennes, France), **Juan Casado-Díaz** (Universidad de Sevilla, Spain).
– **Elisa Davoli** (University of Vienna, Austria), Irene Fonseca (Carnegie Mellon University, USA).
– **Daniela Giachetti** (Università di Roma La Sapienza, Italy).
– Antonio Gaudiello (Universitá Degli Studi Di Napoli Federico II, Italy), **Olivier Guibé** (Université de Rouen Normandie, France), Francois Murat (Laboratoire Jacques-Louis Lions, France).
– **María Eugenia Pérez-Martínez** (Universidad de Cantabria, Spain).
– Carlos Jerez-Hanckes (Universidad Adolfo Ibáñez, Chile), Isabel A. Martínez (Pontificia Universidad Católica de Chile, Chile), **Irina Pettersson** (University of Gävle, Sweden), Volodymyr Rybalko (Institute For Low Temperature Physics and Engineering, Ukraine).
– **Ben Schweizer** (TU Dortmund, Germany).
– **Grigor Nika** (Weierstrass Institute for Applied Analysis and Stochastics, Germany), Bogdan Vernescu (Worcester Polytechnic Institute, USA).

The communications covered a large range of topics, including the influence of a strongly oscillating magnetic field in an elastic body, new microstructure of materials exhibiting interesting, and technologically powerful, elastic and magnetic behaviors, relaxation and homogenization in the framework of A-quasiconvexity for differential operators with coefficients depending on the space variable, problems with weak regularity data involving renormalized solutions, singular nonlinear problems, eigenvalue problems for complicated shapes of the domain, homogenization of partial differential problems with strongly alternating boundary conditions of

Robin type with large parameters, multiscale analysis of the potential action along a neuron with a myelinated axon, dispersive long-time behavior of wave propagation, and multiscale model of magnetorheological suspensions.

It is our desire to conclude this Preface by showing in a special way our thanks to all the authors who agreed to participate in the publication of this volume.

Rouen, France — Patrizia Donato
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