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Stability Analysis of Neural Networks

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This book is dedicated to the authors' beloved family members and all other loved ones as well as to those who have helped the authors' advancements and achievements throughout the years.

Preface

Stability analysis is an important concern for many dynamical systems, and it has been a popular research subject in many fields, including mathematics, control theory, mechanics, and artificial intelligence. Despite its long-standing research history, the issues and challenges of stability analysis with respect to dynamical systems continue to attract the attention of many researchers and scientists over the years. Since the 1990s, investigations on stability analysis pertaining to dynamical systems and networks have progressed rapidly. Retrospectively, the last decade has witnessed many studies related to stability analysis of neural networks, which are formulated as mathematical models that imitate the behaviors of biological nervous systems, in both continuous- and discrete-time domains. Nevertheless, it is still one of the most challenging research topics today due to the absence of complete and comprehensive analytical solutions.

Research in neural networks covers a broad spectrum, and many fruitful real-world applications of neural networks in various areas such as image processing, pattern recognition, and optimization have been reported. The success of these applications largely hinges on the stability characteristics of the equilibrium point from the designed solutions with neural networks. In general, there are two types of stability: asymptotic stability and exponential stability. In this regard, a dominant development of stability analysis of neural networks has been in the broad areas of control and systems engineering. An extensive understanding of these two types of stability analysis is crucial to ensure effective and safe use of neural networks in a variety of practice environments.

For many years, the fundamental methodology for stability analysis of neural networks has been on the use of Lyapunov functionals, and many comprehensive results have been obtained. We do not intend to deliberate on this methodology in an in-depth manner here since it has been well covered in several excellent books. On the other hand, many innovative approaches to stability analysis of neural networks have appeared recently. Consequently, our studies focus on the recent research achievements in innovative stability analysis formulations corresponding to neural networks.

The aim of this book is to provide a strong foundation on recent advances in the stability analysis of various neural network models with constrained signals as well as to highlight the relevant key directions for further research and development. An introductory chapter is first presented to provide an overview of dynamical systems and neural networks, as well as their relevance to the content of this book. We then divide our discussion on stability analysis of neural networks into two parts. Part I, which contains seven chapters, concentrates on the recent stability problems on various continuous-time models of neural networks. Part II, which contains five chapters, focuses on recent stability problems on various discrete-time models of neural networks.

The book serves as an up-to-date reference for graduate students, researchers, and practitioners who are working on, or interested in, the general themes of stability theory, property, and representation of dynamical systems and neural networks, control and systems engineering, and related subjects. It is envisaged that readers can gain a comprehensive understanding and knowledge pertaining to recent advances in a variety of stability issues, challenges, and solutions of continuous-time and discrete-time neural network models through the concepts, principles, and illustrative examples presented throughout the book.

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In fact, such developments are not solely ours, but because we get inspiration and ideas from the work of many friends, seniors, and eminent scholars. Their scientific dedication and especially their influence on our research lead to writing this book in 2020. In this regard, it is a great pleasure to be able to thank many people who have contributed to creating the content of this book. Without their role and help, the publication of this book would not have been possible.

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