Modeling and Simulation in Science,
Engineering and Technology

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David Bakstein

An Introduction to Continuous-Time Stochastic Processes

Theory, Models, and Applications to Finance, Biology, and Medicine

Fourth Edition

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Foreword

Stochastic processes and stochastic calculus have been used extensively for data analysis in a wide range of scientific disciplines. They provide fundamental tools for the construction of stochastic models for datasets under analysis and for statistical inference, including the calculation of uncertainties of estimates and predictors.

Application of stochastic process-based statistical methods (stochastic methods for short) requires a conceptual understanding of the probability theory so as to avoid their misuse. Textbooks for teaching these statistical methods at the university level need to cover the underlying probability theory and stochastic calculus of the statistical methods. The authors have done a remarkable job covering these topics.

This book serves dual purposes of being a textbook for students with a sufficient mathematical background as well as a valuable reference book for preparing mathematicians and professionals in other fields who wish to use stochastic methods.

Part I of the book is a systematic introduction to the theories of stochastic processes and stochastic calculus with an emphasis on concepts that are central to applications. The theoretical introduction is presented in a unifying manner with mathematical clarity. In Part II, numerous examples of applications are given from the fields of finance, biology, epidemics and medicine, as well as from the authors' own work. It conveys the idea that similar stochastic methods have broad applications across different fields. This is in contrast with many other books on stochastic methods that specialize in a particular field of applications.

The book has been well received since the publication of the first edition in 2005, as evidenced by the upcoming publication of the 4th edition. Synchronized with the rapidly expanding application of stochastic processes and an increasing complexity of datasets, the 4th edition incorporates many recent developments in the subject.

It is my great pleasure to write the foreword for the 4th edition of the book. I have known Prof. Vincenzo Capasso since 1972 when he attended my summer school class for graduate assistants/lecturers at Scuola Normale Superiore di Pisa, Italy. The summer school was organized by the late Prof. Edoardo Vesentini, who passed away recently on March 28 2020.

Enzo subsequently visited our department at the University of Maryland during a number of non-consecutive semesters. His first publication in stochastic processes in 1976 has become a widely cited paper on modelling an epidemic by branching processes. The model is based on the Neyman and Scott epidemic model, which was a paper that I gave Enzo to study in the summer school class. I am very pleased to see Enzo's research interest shifted from applied mathematics and physics to stochastic calculus and statistics.

I would place Enzo in Neyman's school of statistics. As one can see from the outset (first edition), the authors state: "This book ... is neither a tract nor a recipe book as such" which reflects Neyman's belief that more mathematical preparedness and understanding of the probability theory is necessary for statistical education, while a "tool box" approach to teaching statistics is not an appropriate choice.

September 2020

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Preface to the Fourth Edition

This fourth edition contains a thorough revision of its predecessors. Firstly, by correcting detected misprints and errors, for which both colleagues and students deserve gratitude. Secondly, yet foremost, additional material has been included on applications of stochastic calculus from advances in recent literature. In particular, the role of random noise in biomedical models is examined in more detail in Chap. 7. There, a diffusion approximation has been included leading to the so-called demographic stochasticity in biomedical models. As a complement, there is a discussion on environmental noise, in particular evidencing paradoxes that may arise when adding Gaussian white noise to parameters. In line with recent literature, models of bounded noise are proposed and for that reason, Chap. 2 now includes a more rigorous introduction to Gaussian white noise, based on the theory of stochastic generalized functions (distributions). Chapter 5 has been thoroughly revised: more material has been added on the stability of stochastic semigroups, which are used in models of population dynamics and epidemic systems. Methods of analysis of one-dimensional stochastic differential equations have been expanded, with particular focus on the existence of invariant distributions. Various new examples and exercises have been added throughout the volume in order to guide the reader through the applications of the theory. Again, in that respect Chap. 7 has been significantly expanded with additional models of population dynamics and epidemiology. For example a nontrivial model of tumor-driven angiogenesis has been added, as an example of a multi-scale system with a stochastic geometric structure. Needless to say that the bibliography has been both updated and extended significantly.

VC would like to acknowledge the original coauthor David Bakstein for his comments to this latest edition.

We are very grateful to all those who have helped us to identify misprints, outright errors and improvements. We are in particular very grateful to Marcello De Giosa, Daniela Morale, Radosław Wieczorek, Grace L. Yang, and to the many students over the years. Discussions with Ryszard Rudnicki and Marta Tyran-Kamińska have been especially important for the new material in Chaps. 7 and 5. Furthermore VC wishes to acknowledge the relevant discussions with Alberto D'Onofrio concerning the crucial role of bounded noise in biomedical models.

Nicola Bellomo, Editor in Chief of this Birkhäuser volume series, is owed a warm acknowledgment for his support since the first edition of this book. More recently, Christofer Tominich, Birkhäuser—Springer Editor, deserves a "thank you" for supporting the publication of this fourth edition and his continuous assistance during all phases of the editorial process.

VC dedicates the 4th Edition of this monograph to his wife Rossana, for her continuous personal support, and to their grandsons Damian and Leonardo (twins).

Milan, Italy Vincenzo Capasso

Preface to the Third Edition

In this third edition, we have included additional material for use in modern applications of stochastic calculus in finance and biology; in particular, Chap. 5 on stability and ergodicity is completely new. We have thought that this is an important addition for all those who use stochastic models in their applications.

The sections on infinitely divisible distributions and stable laws in Chap. 1, random measures, and Lévy processes in Chap. 2, Itô–Lévy calculus in Chap. 3, and Chap. 4, have been completely revisited.

Fractional calculus has gained a significant additional room, as requested by various applications.

The Karhunen-Loève expansion has been added in Chap. 2, as a useful mathematical tool for dealing with stochastic processes in statistics and in numerical analysis.

Various new examples and exercises have been added throughout the volume in order to guide the reader in the applications of the theory. The bibliography has been updated and significantly extended.

We have also made an effort to improve the presentation of parts already included in the previous editions, and we have corrected various misprints and errors made aware of by colleagues and students during class use of the book in the intervening years.

We are very grateful to all those who helped us in detecting them and suggested possible improvements. We are very grateful to Giacomo Aletti, Enea Bongiorno, Daniela Morale, and the many students for checking the final proofs and suggesting valuable changes. Among these, the Ph.D. students Stefano Belloni and Sven Stodtmann at the University of Heidelberg deserve particular credit. Kevin Payne and (as usual) Livio Pizzocchero have been precious for bibliographical references, and advice.

Enea Bongiorno deserves once again special mention for his accurate final editing of the book.

We wish to pay our gratitude to Avner Friedman for having allowed us to grasp many concepts and ideas, if not pieces, from his vast volume of publications.

Allen Mann from Birkhäuser in New York deserves acknowledgment for encouraging the preparation of this third edition.

Last but not the least, we acknowledge the precious editorial work of the many (without specific names) at Birkhäuser, who have participated in the preparation of the book.

Most of the preparation of this third edition has been carried out during the stays of VC at the Heidelberg University (which he wishes to acknowledge for support by BIOMS, IWR, and the local HGS), and at the "Carlos III" University in Madrid (which he wishes to thank for having offered him a Chair of Excellence there).

Milan, Italy

Vincenzo Capasso
David Bakstein

Preface to the Second Edition

In this second edition, we have included additional material for use in modern applications of stochastic calculus in finance and biology; in particular, the section on infinitely divisible distributions and stable laws in Chap. 1, Lévy processes in Chap. 2, the Itô–Lévy calculus in Chap. 3, and Chap. 4. Finally, a new appendix has been added that includes basic facts about semigroups of linear operators.

We have also made an effort to improve the presentation of parts already included in the first edition, and we have corrected the misprints and errors we have been made aware of by colleagues and students during class use of the book in the intervening years. We are very grateful to all those who helped us in detecting them and suggested possible improvements. We are very grateful to Giacomo Aletti, Enea Bongiorno, Daniela Morale, Stefania Ugolini, and Elena Villa for checking the final proofs and suggesting valuable changes.

Enea Bongiorno deserves special mention for his accurate editing of the book as you now see it.

Tom Grasso from Birkhäuser deserves acknowledgement for encouraging the preparation of a second, updated edition.

Milan, Italy

Vincenzo Capasso
David Bakstein

Preface to the First Edition

This book is a systematic, rigorous, and self-contained introduction to the theory of continuous-time stochastic processes. But it is neither a tract nor a recipe book as such; rather, it is an account of fundamental concepts as they appear in relevant modern applications and the literature. We make no pretense of its being complete. Indeed, we have omitted many results that we feel are not directly related to the main theme or that are available in easily accessible sources. Readers interested in the historical development of the subject cannot ignore the volume edited by Wax (1954).

Proofs are often omitted as technicalities might distract the reader from a conceptual approach. They are produced whenever they might serve as a guide to the introduction of new concepts and methods to the applications; otherwise, explicit references to standard literature are provided. A mathematically oriented student may find it interesting to consider proofs as exercises.

The scope of the book is profoundly educational, related to modeling real-world problems with stochastic methods. The reader becomes critically aware of the concepts involved in current applied literature and is, moreover, provided with a firm foundation of mathematical techniques. Intuition is always supported by mathematical rigor.

Our book addresses three main groups of readers: first, mathematicians working in a different field; second, other scientists and professionals from a business or academic background; third, graduate or advanced undergraduate students of a quantitative subject related to stochastic theory or applications.

As stochastic processes (compared to other branches of mathematics) are relatively new, yet increasingly popular in terms of current research output and applications, many pure as well as applied deterministic mathematicians have become interested in learning about the fundamentals of stochastic theory and modern applications. This book is written in a language that both groups will understand and in its content and structure will allow them to learn the essentials profoundly and in a time-efficient manner. Other scientist-practitioners and academics from fields like finance, biology, and medicine might be very familiar with a less mathematical approach to their specific fields and thus be interested in learning the mathematical techniques of modeling their applications.

Furthermore, this book would be suitable as a textbook accompanying a graduate or advanced undergraduate course or as secondary reading for students of mathematical or computational sciences. The book has evolved from course material that has already been tested for many years in various courses in engineering, biomathematics, industrial mathematics, and mathematical finance.

Last, but certainly not least, this book should also appeal to anyone who would like to learn about the mathematics of stochastic processes. The reader will see that previous exposure to probability, though helpful, is not essential and that the fundamentals of measure and integration are provided in a self-contained way. Only familiarity with calculus and some analysis is required.

The book is divided into three main parts. In Part I, comprising Chaps. 1–4, we introduce the foundations of the mathematical theory of stochastic processes and stochastic calculus, thereby providing the tools and methods needed in Part II (Chaps. 6 and 7), which is dedicated to major scientific areas of application. The third part consists of appendices, each of which gives a basic introduction to a particular field of fundamental mathematics (e.g., measure, integration, metric spaces) and explains certain problems in greater depth (e.g., stability of ODEs) than would be appropriate in the main part of the text.

In Chap. 1 the fundamentals of probability are provided following a standard approach based on Lebesgue measure theory due to Kolmogorov. Here the guiding textbook on the subject is the excellent monograph by Métivier (1968). Basic concepts from Lebesgue measure theory are also provided in Appendix A.

Chapter 2 gives an introduction to the mathematical theory of stochastic processes in continuous time, including basic definitions and theorems on processes with independent increments, martingales, and Markov processes. The two fundamental classes of processes, Poisson and Wiener, are introduced as well as the larger, more general, class of Lévy processes. Further, a significant introduction to marked point processes is also given as a support for the analysis of relevant applications.

Chapter 3 is based on Itô theory. We define the Itô integral, some fundamental results of Itô calculus, and stochastic differentials including Itô's formula, as well as related results like the martingale representation theorem.

Chapter 4 is devoted to the analysis of stochastic differential equations driven by Wiener processes and Itô diffusions and demonstrates the connections with partial differential equations of second order, via Dynkin and Feynman–Kac formulas.

Chapter 6 is dedicated to financial applications. It covers the core economic concept of arbitrage-free markets and shows the connection with martingales and Girsanov's theorem. It explains the standard Black–Scholes theory and relates it to Kolmogorov's partial differential equations and the Feynman–Kac formula. Furthermore, extensions and variations of the standard theory are discussed as are interest rate models and insurance mathematics.

Chapter 7 presents fundamental models of population dynamics such as birth and death processes. Furthermore, it deals with an area of important modern research—the fundamentals of self-organizing systems, in particular focusing on the social behavior of multiagent systems, with some applications to economics ("price herding"). It also includes a particular application to the neurosciences, illustrating the importance of stochastic differential equations driven by both Poisson and Wiener processes.

Problems and additions are proposed at the end of the volume, listed by chapter. In addition to exercises presented in a classical way, problems are proposed as a stimulus for discussing further concepts that might be of interest to the reader. Various sources have been used, including a selection of problems submitted to our students over the years. This is why we can provide only selected references.

The core of this monograph, on Itô calculus, was developed during a series of courses that one of the authors, VC, has been offering at various levels in many universities. That author wishes to acknowledge that the first drafts of the relevant chapters were the outcome of a joint effort by many participating students: Maria Chiarolla, Luigi De Cesare, Marcello De Giosa, Lucia Maddalena, and Rosamaria Mininni, among others. Professor Antonio Fasano is due our thanks for his continuous support, including producing such material as lecture notes within a series that he coordinated.

It was the success of these lecture notes, and the particular enthusiasm of coauthor DB, who produced the first English version (indeed, an unexpected Christmas gift), that has led to an extension of the material up to the present status, including, in particular, a set of relevant and updated applications that reflect the interests of the two authors.

VC would also like to thank his first advisor and teacher, Prof. Grace Yang, who gave him the first rigorous presentation of stochastic processes and mathematical statistics at the University of Maryland at College Park, always referring to real-world applications. DB would like to thank the Meregalli and Silvestri families for their kind logistical help while he was in Milan. He would also like to acknowledge research funding from the EPSRC, ESF, Socrates–Erasmus, and Charterhouse and thank all the people he worked with at OCIAM, University of Oxford, over the years, as this is where he was based when embarking on this project.

The draft of the final volume was carefully read by Giacomo Aletti, Daniela Morale, Alessandra Micheletti, Matteo Ortisi, and Enea Bongiorno (who also took care of the problems and additions) whom we gratefully acknowledge. Still, we are sure that some odd typos and other, hopefully noncrucial, mistakes remain, for which the authors take full responsibility.

We also wish to thank Prof. Nicola Bellomo, editor of the "Modeling and Simulation in Science, Engineering and Technology" series, and Tom Grasso from Birkhäuser for supporting the project. Last but not least, we cannot neglect to thank Rossana (VC) and Casilda (DB) for their patience and great tolerance while coping with their "solitude" during the preparation of this monograph.

Milan, Italy

Vincenzo Capasso
David Bakstein

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