Valérie Girardin
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Applied Probability

From Random Experiments to Random Sequences and Statistics

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Preface

Some of the books on probability that have been published in the past years focus on theoretical developments, while others are oriented towards applications. Books written for beginners in probability, or dedicated to a particular application, usually contain only a limited number of theoretical notions, while others are very complete but very dense. We hope that a pragmatic book close to applications but without giving up mathematical rigor, serious yet friendly, can at the same time be useful and topical.

Random variables are defined by reference to a random experiment as functions whose values depend on the result of the experiment. They are called real random variables if they take values in the real line. A finite family of such variables is a random vector, while a denumerable family is a random sequence. All these random elements are studied together with notions necessary to their use in applied fields. To this end, we consider reliability linked to the lifetime of living and industrial systems, entropy linked to information theory, simulation methods, and we also present basics in inferential statistics.

Even if discrete finite probability spaces and random variables can be studied through elementary methods, the associated theory remains insufficient to model rigorously classical stochastic experiments; for instance, infinite sequences of tossing need a more sophisticated treatment. We have therefore chosen to include the discrete spaces in the more general framework of measured spaces in which all possible real situations are included. Nevertheless, we present measure and integration theory only as much as necessary to understand the spaces in which we actually work.

This book is addressed to advanced undergraduate students in mathematics and to postgraduate students in applied mathematics. It will also be of use to researchers and engineers working in other fields but interested by the rigorous mathematical basis of probability theory. To benefit from the reading, no notions of probability are required. The prerequisites are limited to some classical undergraduate courses in mathematical analysis.

Each chapter is illustrated by a great number of examples and exercises with their solutions. They are intended not only as solutions of classical problems in probability but also as complements to the main text together with an opening to applied fields. A table of notation and a detailed index are given for easy reference. A classified bibliography proposes further reading in theoretical and applied fields.

The volume is organized as follows.

In Chap. 1, the basic notions of Kolmogorov's system of axioms is presented, and completed by all intrinsic notions of probability theory, such as independence and conditioning of events. We state different formulae necessary for the effective calculus of probability of events. Primary principles of entropy end the chapter.

In Chap. 2, the discrete or continuous real random variables are presented, together with the tools necessary to their investigation, from probabilistic tools such as distributions and distribution functions to analytical tools such as moment generating functions of all kinds. First notions of reliability are given.

In Chap. 3, the simultaneous study of several real random variables is presented. The notions given in Chaps. 1 and 2 are extended to these real random vectors. Specific notions linked to relations between variables such as independence, order statistics, and entropy are given. Attention is focused on the effective calculus of distributions of random variables and vectors, in particular, Gaussian vectors.

In Chap. 4, elements of stochastic topology with the different types of convergence of random sequences are presented: almost sure, in mean, in square mean, in probability, and in distribution. We detail different laws of large numbers and central limit theorems, the most remarkable results of probability theory. Some basic stochastic simulation methods are developed.

In Chap. 5, basic notions of parametric and non-parametric inferential statistics are presented. Point estimation, confidence intervals, and statistical testing are a first step in the huge domain of mathematical statistics.

The volume is our own loose translation of the first of our two books published in French by Vuibert, Paris, which are in their third edition. The second volume, published in English by Springer in 2018, begins where this volume ends. The interested reader will find there complements on random sequences indexed by integers—such as martingales and Markov chains, together with an introduction to the random processes theory—especially jump Markov and semi-Markov processes. Application is proposed in many fields of applied probability, such as reliability, information theory, production, risk, seismic analysis, and queueing theory.

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