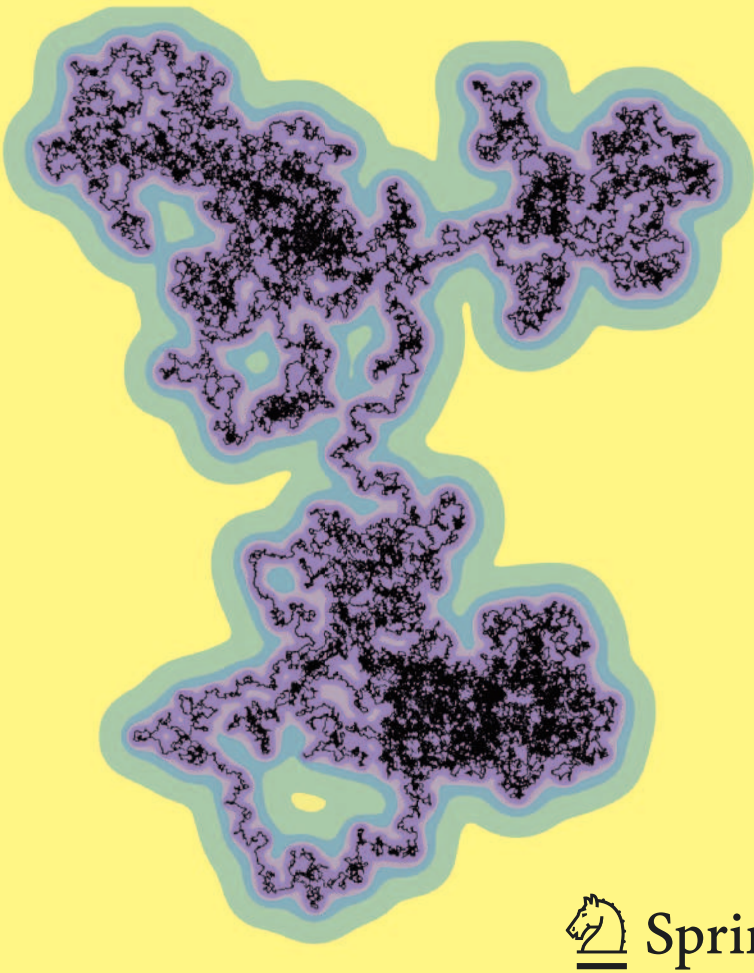
The Mathematics of Errors

Nicolas Bouleau



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Introduction

In probability courses, the teacher often begins with the words "Let ... be a probability space", writing $(\Omega, \mathcal{A}, \mathbb{P})$ in the upper left corner of the blackboard. Then all constructions and reasoning can be conducted, using products to express independence, projections for conditional expectations, convergences to establish the laws of large numbers, etc. Probabilistic models are very rich and appear in many fields with the widely used notions of the Markov chains, stationary processes and diffusion processes. They now play a role in physics, economics and the natural and environmental sciences.

In this treatise, we propose to enrich probability theory by adding to probability measures operators that can accompany them in all constructions and deductions. This is what the *Dirichlet forms* are capable of, which can be thought of as differential calculus tools compatible with probability calculus.

In addition to the inherent interest of this investigation, which has already provided several new results on important topics and still involves open questions, the resulting world of enriched probabilities is exactly what is needed to conduct rigorous error calculations in complex models. It is this interpretation that is chosen here as a guiding thread.

* *

Historically, the theory of the Dirichlet forms was born as an improvement of the Hilbertian methods in potential theory, independently of any interpretation in terms of error calculations, and it became a very active field of research in the 1980s when it was realized that it easily extends to infinite-dimensional spaces. The interpretation as an error calculation is much easier than one might imagine. It allows physicists, engineers and modelers to assimilate these techniques and researchers to rely on a solid intuition.

We present it in a progressive way, by relying on physical intuition and on common uses in matters of errors more or less known to everyone. But this forces us to criticize certain habits, in order to better understand the fertility of a rigorous theory.

The present edition is based on the French book [294] of the author. The work proceeds very gradually, starting from concrete intuition and the culture concerning errors, then introducing the mathematical tools on which the theory is based (semi-groups of operators, Dirichlet forms) and, thanks to them, developing error calculations on complex models such as those involving the Brownian motion.

The *exercises* in the course of the text or at the end of the chapters are not riddles but examples or in-depth studies that can be followed pen in hand and whose details can be completed using the materials of the treatise.

I would like to pay a special tribute here to the mathematicians who enabled me to succeed in writing these ideas, through their teaching. Here, I am thinking in particular of Laurent Schwartz, Gustave Choquet and Jacques Neveu and the dynamism that their research has generated. I also especially want to quote Paul-André Meyer, Masatoshi Fukushima and Paul Malliavin and, finally, for intimate and lasting collaborations, Francis Hirsch, Dominique Lépingle, Christophe Chorro and Laurent Denis, in liaison with the potential theory group in France and the community of practitioners of the Dirichlet forms, mainly in Germany and Japan.

Masatoshi Fukushima and Nicolas Bouleau
Bielefeld, June 2005

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