

Springer Optimization and Its Applications 193

Michael A. Henning
Jan H. van Vuuren

Graph and Network Theory

An Applied Approach using
Mathematica[®]


 Springer

Springer Optimization and Its Applications

Volume 193

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Aims and Scope

Optimization has continued to expand in all directions at an astonishing rate. New algorithmic and theoretical techniques are continually developing and the diffusion into other disciplines is proceeding at a rapid pace, with a spot light on machine learning, artificial intelligence, and quantum computing. Our knowledge of all aspects of the field has grown even more profound. At the same time, one of the most striking trends in optimization is the constantly increasing emphasis on the interdisciplinary nature of the field. Optimization has been a basic tool in areas not limited to applied mathematics, engineering, medicine, economics, computer science, operations research, and other sciences.

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ISSN 1931-6828 ISSN 1931-6836 (electronic)
Springer Optimization and Its Applications
ISBN 978-3-031-03856-3 ISBN 978-3-031-03857-0 (eBook)
<https://doi.org/10.1007/978-3-031-03857-0>

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Dedication

We dedicate this book with love and appreciation to our mothers.



Joyce Henning-Smith



Annalene van Vuuren

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