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Advances in Variable Structure and Sliding Mode Control

With 170 Figures

Springer

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Preface

Variable Structure Control (VSC) and its main mode of operation – Sliding Mode Control (SMC) – is recognized as an efficient tool to design controllers which are robust with respect to uncertainty. The main advantages are:

- low sensitivity to plant parameters and perturbations, eliminating the need for exact modelling;
- the possibility of decoupling the original plant system into two components of lower dimension;
- many controllers ensure finite time convergence to the switching surface and can be straightforwardly implemented.

However the development of the VSC methodology has also exposed disadvantages. Specifically:

- the chattering problem caused by imperfections in sensors, actuators and switching devices and discrete realization;
- the fact that traditionally VSC design focussed on the presence of the matched uncertainties only.

Many of the chapters in this book are based on expansions of selected presentations from the 8th IEEE International Workshop on Variable Structure Systems VSS'04, which was held in Barcelona, Spain in September 2004. The editors have tried to identify the key contributions from this workshop, which define the state of the art, represent new directions building on existing work, and highlight new emerging application areas. In particular, many of the chapters address the two disadvantages indicated above. To ensure fair coverage of recent major contributions, other experts in the field, who were not able to attend VSS'04, have been invited to contribute chapters to the book.

The structure of the book is as follows: the first few chapters present new underpinning mathematical ideas associated with VSC. Next, advances in higher-order sliding are described. SMC for systems with both matched and unmatched uncertainties are then presented. The fourth part is devoted to sliding mode observers and their potential application areas. Finally a broad range of practical implementations and the application of SMC to different engineering fields are described.

The first chapter by Yuri Orlov develops new analysis tools for uncertain VSS' employing non-smooth Lyapunov functions with non-positive time derivatives along the system trajectories. An auxiliary indefinite function is employed in the stability analysis in order to extend Krasovskii-LaSalle's invariance principle to non-autonomous VSS'. The capabilities of the extended invariance principle are illustrated by its application to the problem of position feedback regulation of mechanical manipulators in the presence of significant friction levels. The controller, obtained by applying the extended invariance principle, proves to be capable of providing the desired system performance in spite of complex hard-to-model nonlinear friction phenomena.

Chapter 2, by Emmanuel Moulay and Wilfrid Perruquetti, presents state-of-the-art results in the area of finite-time stability and stabilization. This chapter is divided into four main themes: a) definitions and analysis finite-time stability of continuous systems; b) a collection of necessary and/or sufficient conditions for testing this property; c) the extension of these ideas to finite-time stability for discontinuous systems; d) finally the finite-time stabilization problem is considered.

In Chapter 3 by Giorgio Bartolini, Franco Parodi, Elisabetta Punta and Tullio Zolezzi, a multi-input sliding mode control technique, called the simplex method, is presented. Multi-input sliding mode control for nonlinear uncertain systems is not a simple generalization of standard single input VSC approach, except in the case in which the uncertainties, though affecting the system model, do not appear in the control matrix. This chapter describes the development of simplex methods, originated by the pioneering work of Baida and Izosimov. Two kinds of simplex control methods are studied. These methods are analyzed in a more general framework than previously. Finite time convergence results are obtained, under explicitly computable conditions, for multi-input multi-output nonlinear control systems with deterministic uncertainties. The control design is based on the knowledge only of the nominal plant and the convergence conditions make use of the available bounds of the uncertain dynamics. Special attention is devoted to plants affine in the control law in order to show the applicability of the proposed control methods to complex uncertain systems such as a robotic structure.

Chapter 4 by Fabiola Angulo, Mario di Bernardo and Gerard Olivar brings together results from the theory of non-smooth bifurcations and control. Several strategies are presented showing how a discontinuous control strategy can modify the characteristics of a target limit cycle. Some of the strategies presented in the chapter take advantage of specific sliding trajectories. Control is achieved by modifying the corresponding Poincare map. As an illustrative example, the normal form of a Hopf bifurcation has been used for clarity of presentation.

Chapter 5 by Yaodong Pan and Katsuhisa Furuta is concerned with VSC using sectors for the switching rule design. A sliding sector is a subset of the state space, inside which a norm of the state decreases without any control action. Originally, the sliding sector was proposed to replace the sliding mode for a chattering free VS control and for the implementation of the VS control in discrete-time control systems. In this chapter, a system with a set of available control laws is considered. A sliding sector is defined for the system with each available control law. Then a VS control system is designed to switch the system input among those available control laws such that the system is always inside one of the sliding sectors with its corresponding control law and some Lyapunov function keeps decreasing with no requirement of converging to any sliding mode or to any sliding boundary layer, or to any sliding sector.

In Chapter 6, by Hoon Lee and Vadim Utkin, the chattering phenomenon, caused by the presence of unmodelled dynamics, is revisited. The authors show that if the relative degree of the complete model with respect to the sliding surface is more than two, then chattering occurs. For analysis of the amplitude and the frequency of the chattering, the describing function method is used. The authors go on to discuss the properties of two principal approaches for chattering suppression: the observer based solution and relay control gain adaptation.

The second part of the book is concerned with the emerging area of Higher Order Sliding modes. In Chapter 7, written by Igor Boiko and Leonid Fridman, a number of second order sliding mode (SOSM) control algorithms are analyzed in the frequency domain using describing function methods. It is shown that in the presence of parasitic dynamics the system driven by a SOSM algorithm always exhibits chattering. The mechanism of chattering generation is analyzed and it is argued that chattering is an inherent feature of the sliding mode principle, and therefore unavoidable even when second order sliding mode algorithms are continuous. In addition a method for comparison of chattering parameters for different second order sliding mode algorithms is suggested.

In Chapter 8, written by Arie Levant, a general uncertain SISO tracking problem with a well-defined relative degree r is shown to feature homogeneity, provided the controller possesses some specific homogeneity called r-sliding homogeneity. In particular, the tracking accuracy of such controllers is proportional to the sampling noise magnitude. Any such controller can be complemented by a (r-1)th-order homogeneous differentiator preserving its stability and accuracy properties. As an application of this theory, quasi-continuous controllers are developed. These controllers lose their continuity only when the zero-dynamics (r-sliding mode) is maintained. In practice, the corresponding r equalities are never fulfilled simultaneously due to the presence of noises and imperfections, and hence the control remains continuous for all time. As a result it is natural to expect better performance and less chattering in that case, which is confirmed by simulation.

In Chapter 9 by Salah Laghrouche, Franck Plestan and Alain Glumineau, a new methodology for the design of a robust VSC is presented. This control law ensures the establishment of (practical) higher order sliding mode behaviour with classical features such as robustness and finite-time convergence. The approach is based in part on using LQ control for the design of the switching variable. The main advantages of the approach are, a-priori knowledge of the convergence time and simplicity of implementation. The scheme is demonstrated on two examples: the control of a synchronous motor and control of a biped walking robot.

In Chapter 10 the problem of controlling uncertain nonlinear multi-input-multi-output systems using output-feedback is addressed by Liu Hsu, Alessandro Jacoud Peixoto, José Paulo V. S. Cunha, Ramon R. Costa and Fernando Lizarralde. A model-reference sliding mode control approach is adopted to develop a scheme applicable to plants with arbitrary uniform relative degree. Nonlinearities of a given class are incorporated as state dependent and possibly unmatched disturbances to a linear plant. No particular growth conditions are imposed on the output dependent nonlinearities. In contrast to high-gain observer based schemes, no explicit state observers are employed and the control signal is free of peaking. Prior knowledge about the high frequency gain matrix is reduced to a Hurwitz condition and is less restrictive than the conditions in other existing approaches. Global or semi-global exponential stability with respect to some small residual set is established.

In Chapter 11, Fernando Castanos, Jian-Xin Xu and Leonid Fridman, study the robustness properties of integral sliding mode controllers with respect to both matched and unmatched uncertainties. The chapter shows how to select the projection matrix in such a way that the Euclidean norm of the resulting perturbation is minimal. It is also shown that when the minimum is attained, the resulting perturbation is not amplified. This selection is particularly useful if integral sliding mode control is to be combined with other methods to further 'robustify' against unmatched perturbations.

Chapter 12 by Leonid Fridman, Alex Poznyak, Yuri Shtessel and Franciso Bejarano continues the investigation of the robustness properties of integral and conventional sliding modes for a linear time invariant min-max multi-model problem with uncertainties. The proposed modified integral sliding mode dynamics enable the dimension of the min-max control design problem to be reduced from [Nn] to the space of unmatched uncertainties which is [Nn-(N-1)m]. A combined 'robustifying' strategy obtained by designing the optimal sliding mode and ensuring robustness of the trajectories during the reaching phase via integral sliding modes is suggested. A numerical example illustrates that the suggested modification of the integral sliding mode dynamics does not change the optimal control and the value of the corresponding performance index.

The third part of the book is concerned with observers and problems of system identification. The chapter by Thierry Floquet and Jean-Pierre Barbot introduces a constructive algorithm that transforms a certain class of linear systems with unknown inputs into a canonical form suitable for the design of finite time sliding mode observers. An important property of this scheme is that it is possible to estimate in finite time the state and the unknown inputs of systems that do not necessarily satisfy the classical matching conditions involved for the design of sliding mode observers. In this chapter, the step-by-step observer is derived under second order sliding mode considerations and without the requirement of low pass filters.

Chapter 14, written by Christopher Edwards and Chee Pin Tan, describes the implementation of a recently proposed robust sliding mode observer-based FDI scheme. The scheme is applicable to both actuator and sensor faults. The novelty of the scheme is that fault signals are reconstructed and that the effect of the uncertainty present in the model to encapsulate the plant model/mismatch, is minimized in an $\mathcal{L}_2$ sense. The design approach yields a convex optimization problem which can be efficiently solved using standard LMI software. One advantage of this approach is that because faults are reconstructed, these signals can be used to correct a faulty sensor for example, and maintain reasonable performance until appropriate maintenance could be undertaken. This 'virtual sensor' can be used in the control algorithm to form the output tracking error signal which is processed to generate the control signal. This idea has been implemented successfully on a dc-motor rig. The scheme is not specific to such a system and is applicable to a reasonably wide class of systems which can, at least in an operating region of interest, be adequately represented by an uncertain linear system.

In Chapter 15, by Alex Poznyak, Yuri Shtessel, Leonid Fridman, Jorge Davila and Jesica Escobar new approaches to solve identification problems based on sliding mode observers are given. It is shown that the value of the equivalent output error injection identifies unknown inputs directly. Continuous time versions of classical Least Squares and Forgetting Factor Methods are modified to identify unknown time-invariant and time-varying parameters using equivalent output error injection ideas.

In 'Discrete-time Sliding Mode Control Using Multirate Output Feedback' the authors B. Bandyopadhyay and S. Janardhanan present some algorithms for robust sliding mode control of discrete-time linear systems with bounded uncertainty using the concept of multirate output feedback, in which the sensor output and actuator signal are sampled at different rates. With the increasing use of computers for controller implementation, discrete controllers and system analysis in the discrete-time domain have gained importance. Moreover, the study of output feedback sliding mode control technique has a greater practical value in comparison with state feedback sliding mode control techniques.

The final part of the book is devoted to a collection of applications of sliding mode control in a variety of situations. In Chapter 17, Nitin Patel, Christopher Edwards and Sarah Spurgeon consider a sliding mode based scheme for optimal deceleration in an automotive braking manoeuvre. The scheme is model based and seeks to maintain the longitudinal slip value associated with the tyre road contact patch at an optimum value – the point at which the friction coefficient-slip curve reaches a maximum. The scheme assumes only wheel angular velocity is measured, and uses a sliding mode observer to reconstruct the states and a road conditions parameter for use in the controller. The sliding mode controller then seeks to maintain the vehicle at this optimal slip value through an appropriate choice of sliding surface.

Chapter 18, by Asif Sabanovic, Khalid Abidi and Çağdaş Önal describes the application of sliding mode control methods to the problem of high precision actuation based on piezoelectric stack actuators. Both position tracking and force control are discussed. In order to achieve high accuracy in position control a SMC controller is combined with a disturbance observer. The SMC disturbance observer is based on a lumped parameter model of the actuator and allows nonlinear hysteresis disturbances, external forces as well as parameter variations to be estimated. For force control the same structure of observer is used for external force estimation, which by using a nonlinear differential equation, allows the hysteresis nonlinearity to be removed from the total disturbance. It is then possible to use this estimate of external force in an observer based force controller for the actuator. Simulation and experimental results are compared to validate the disturbance and external force estimation technique. Experiments that incorporate disturbance compensation within a closed-loop SMC control algorithm are also presented to prove the effectiveness of this method in producing high precision motion.

In Chapter 19, Giorgio Bartolini, Nicola Orani, Alessandro Pisano and Elio Usai, describe some recent results concerning the application of higher-order sliding-mode techniques to control and estimation problems in the area of electrical drives. Both induction and direct-current permanent-magnet motors are dealt with. For Induction Motor drives, the important problems of motion control and rotor resistance identification are addressed and solved by means of a combination of adaptive and sliding mode control concepts. The control problem for DC motors with uncertain parameters and unknown external loads is solved by means of an output feedback controller which combines higher-order sliding mode controllers and higher-order sliding mode differentiators. Detailed experimental analysis is provided.

Chapter 20, written by Alan Zinober, Yuri Shtessel, Enric Fossas, Josep Olm and Joe Patterson describes the application of SMC to the control of switched mode DC-to-DC power converters in tracking a real-time voltage profile. This is very promising because a switching control strategy is traditionally employed in power converters, and because of the inherent robustness properties of the sliding mode. Direct regulation/tracking control of the output voltage for boost and buck-boost power converters results in a nonminimum phase system and therefore an unstable controller. The analysis and control of multiple modular DC-to-DC boost power converters connected in parallel sharing a common voltage source, that includes the source resistance, is studied. Source resistance should be considered because of its power limiting and coupling effects. The method of stable system centre is employed and results in high fidelity output voltage tracking and decoupling for this nonminimum phase system using *direct* and *indirect* control. The first approach is based on the transformation of nonminimum phase output tracking in causal systems to state variable tracking. The bounded state reference profiles are generated using custom-designed equations of the system centre. The second technique propounds an indirect control of the output voltage through the input current. Disturbance effects are rejected by means of a dynamic compensator based on an analytical approximation of a bounded solution to the unstable internal dynamics, obtained by means of the Galerkin method.

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