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Advanced Strategies in Control Systems with Input and Output Constraints

Springer

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Preface

Physical, safety or technological constraints induce that the control actuators can neither provide unlimited amplitude signals nor unlimited speed of reaction. The control problems of combat aircraft prototypes and satellite launchers offer interesting examples of the difficulties due to these major constraints. Neglecting actuator saturations on both amplitude and dynamics can be source of undesirable or even catastrophic behavior for the closed-loop system (such as loosing closed-loop stability) [3]. Such actuator saturations have also been blamed as one of several unfortunate mishaps leading to the 1986 Chernobyl nuclear power plant disaster [12], [10]. For these reasons, the study of the control problem (its structure, performance and stability analysis) for systems subject to both amplitude and rate actuator saturations as typical input constraints has received the attention of many researchers in the last years (see, for example, [13], [8], [7], [6]).

Anti-windup is an empirical approach to cope with nonlinear effects due to input constraints, and override is a related technique for handling output constraints, [6].

The anti-windup approach consists of taking into account the effect of saturations in a second step after a previous design performed disregarding the saturation terms. The idea is then to introduce control modifications in order to recover, as much as possible, the performance induced by a previous design carried out on the basis of the unsaturated system. In particular, anti-windup schemes have been successfully applied to avoid or minimize the windup of the integral action in PID controllers. This technique is largely applied in industry. In this case, most of the related literature focuses on the performance improvement in the sense of avoiding large and oscillatory transient responses (see, among others, [1], [5]).

More recently, special attention has been paid to the influence of the anti-windup schemes on the stability and the performance of the closed-loop system (see, for example, [2], [9], [11]). Several results on the anti-windup problem are concerned with achieving global stability properties. Since global results cannot be achieved for open-loop exponentially unstable linear systems in the presence of actuator saturation, local results have to be developed. In this context, a key issue concerns the determination of stability domains for the closed-loop system. If the resulting basin of attraction is not sufficiently large, the system can present a divergent behavior depending on its initialization and the action of disturbances. It is worth to notice that the basin of attraction is modified (and therefore can be enlarged) by the anti-windup loop. In [4], or in the ACC03 Workshop "T-1: Modern Anti-windup Synthesis", some constructive conditions are proposed both to determine suitable anti-windup gains and to quantify the closed-loop region of stability in the case of amplitude saturation actuator.

The override technique uses the same basic approach of a two-step design. A linear control loop is designed for the main output first without regard of the output constraints. It normally performs control for small enough deviations form its design operating point. Then one or more additional feedback control loops are designed for the system trajectory to run along or close to those output constraints. The transfer between the loops is automatic (for example by Min-Max-Selectors) and bumpless (by using antiwindup) and constitutes the dominant nonlinear element in the control system, Much less research results have been published on this topic so far, [6].

The book is organized as follows.

- Part 1 is devoted to anti-windup strategies and consists of chapters 1 through 6.
- Part 2 is devoted to model predictive control (MPC) and consists of chapters 7 through 10.
- Part 3 is devoted to stability and stabilization methods for constrained systems and consists of chapters 11 through 15.

Note that this partition is somewhat arbitrary as most of the chapters are interconnected, and mainly reflects the editors' biases and interests.

We hope that this volume will help in claiming many of the problems for controls researchers, and to alert graduate students to the many interesting ideas

Proposal of Benchmarks (Common Application Examples)

We decided to provide two benchmark problems, hoping that this will make reading the book more attractive. One of them or both are considered in several chapters. However the given benchmark problems may not be ideally suited to each design method. Then in some chapters, an additional case has been supplemented.

Both examples are abstracted from their specific industrial background, but conserve the main features which are relevant in this specific context.

The main focus is on plants which are exponentially unstable systems. They are known to be sensitive to constraints. We elected this feature as it seems currently to be the most interesting and also the most challenging one. Hurwitz systems have been covered in many publications in recent years, and that area seems to have matured.

In operation, both plants have a strong persistent disturbance z, which moves the steady state value $\bar{u}$ of the control variable u close to the respective saturation. As one control engineer put it: ‘operate as close as possible to the top of the hill (to maximize revenue), but do not fall off the cliff on the other side’.

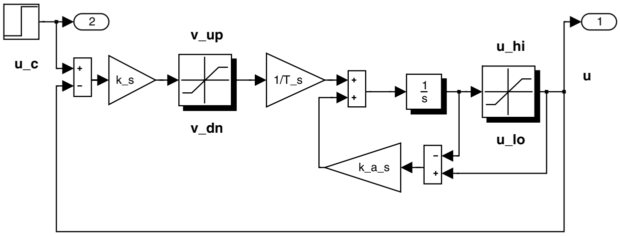
The plants are described by models in continuous form. Model variables are scaled and given in ‘per unit’ form. All state variables x_i are measured and are available as outputs y_i. No provision has been made for fast non-modelled dynamics. However it is recommended to check for possible bandwidth limitations. The same holds for the inevitable measurement noise.

The controllers may be implemented in continuous or time discrete form.

Integral action or a suitable substitute shall be provided, in order to drive the control error to zero for steady state loads $\bar{z} \neq 0$.

A typical electro-hydraulic actuator subsystem is considered. The mechanical end stops of the servomotor piston are at u_{lo}, u_{hi} (*stroke constraints*), and v_{lo}, v_{hi} represent the flow constraints from the pilot servovalve (*slew constraints*), and k_s is the finite, moderate gain of the P-controller.
This model is embedded in the typical cascade structure, where the main controller outputs the position reference $u_c = r_s$.

In the contribution we expect the contributor to



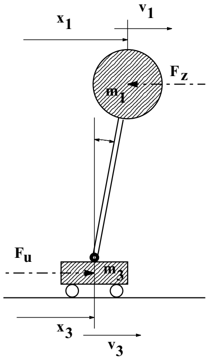
- show that the specifications are met,
- document the system responses to a given test sequence, and indicate the maximum stabilizable inputs,
- provide a stability analysis,
- and optionally to comment on the implementation requirements.

The Inverted Pendulum

Consider the inverted pendulum around its upright position for small inclination angles. The pendulum mass m_1 shall be concentrated at its center of gravity (cg), and the connecting member of length L to the slider at its bottom is mass-free and rigid. The slider mass m_3 may move horizontally without friction. A horizontal force is applied by the actuator subsystem to the slider, in order to control the speed error to zero. Alternatively you may investigate the case of driving the position error to zero. Denote as state variables the horizontal speed of the pendulum cg as x_1, the horizontal displacement between the cg's of pendulum and slider as x_2 and the horizontal speed of the slider as x_3.

The load z_1 represents a horizontal force on the pendulum cg, which is persistent and with unknown but bounded magnitude. And u represents the force by the actuator subsystem, which is constrained first in magnitude (stroke) only and then in rate (slew) as well.

- The linearized model in appropriately scaled variables is given by



$$\tau_1 \frac{d}{dt} x_1 = -x_2 + z_1;\ y_1 = x_1$$
$$\tau_2 \frac{d}{dt} x_2 = -x_1 + x_3;\ y_2 = x_2$$
$$\tau_3 \frac{d}{dt} x_3 = +x_2 + u;\ y_3 = x_3$$

- with parameter values $\tau_3 = \tau_2 = \tau_1 = 1.0$ s.
- The transfer functions have one pole at the origin, one on the negative real axis, and one symmetrical on the positive real axis.
- The closed loop bandwidth in the linear range is specified at ≥ 2.0 rad/s.
- The actuator magnitude (stroke) saturations are at

  $u_{lo} = -1.25$ and $u_{hi} = +1.25$

  and on the actuator rate (slew) are:

  (a) $|du/dt| \leq 10.0$/s and then (b) $|du/dt| \leq 2.0$/s
- The test sequence is defined as follows
  - always start in closed loop operation at initial conditions $\mathbf{x}(0) = \mathbf{0}$, reference $r_1 = 0$ and load $z_1 = 0$.
  - apply a large reference ‘step’ of size r_1
    up to max. stabilizable or to $r_1 = 2.0$, whichever is smaller,
    with slew rate $dr_1/dt = +0.5$/s,

    after stabilization apply a small additional reference $\Delta r_1 = 0.10$
    and then back to $\Delta r_1 = 0.0$.

    and set back $r_1 = 0.$, with slew rate $dr_1/dt = -0.5$/s

- apply a large load 'step' of size z_1
  up to $z_1 = 1.0$ or to max. stabilizable, whichever is smaller
  with slew rate $dz_1/dt = +0.25$/s

  after stabilization apply a small additional load 'step' $\Delta z_1 = 0.10$
  and then back to $\Delta z_1 = 0.0$.

  and set back $z_1 = 0.$, with slew rate $dz_1/dt = -0.25$/s

□

Continuous Stirred Tank Reactor (CSTR) with a Strong Exothermal Reaction

Consider a batch reactor with a continuous feed flow of reactant and control of the contents temperature by means of a heating/cooling jacket.

The main variable to be controlled is the fluid temperature x_2, to $r_2 = 1.0$, with zero steady state error $e_2 = r_2 - x_2 \rightarrow 0$.

The main disturbance to the temperature control loop is the thermal power production. It should be as high as possible without the temperature x_2 running away, in order to maximize the production rate. The limit to this is set by the maximum cooling heat flow though the wall to the jacket, and thus by the minimum temperature of the heating/cooling fluid (u_2) entering the jacket. Both magnitude and rate constraints on u_2 are specified from the subsystem delivering the heating/cooling fluid.

The reactant concentration x_1 is to be controlled to its setpoint r_1 through the reactant feed flow u_1, with no perceptible magnitude or rate constraints here.

Both the temperature controller R_2 and the concentration controller R_1 are to be designed.

The model of the plant shall be given by the nonlinear equations (in 'per-units'):

$$\tau_1 \tfrac{d}{dt} x_1 = -a_1\, f \qquad + u_1; \qquad y_1 = x_1$$

$$\tau_2 \tfrac{d}{dt} x_2 = -a_2\, f \qquad + a_3(x_3 - x_2); \quad y_2 = x_2$$

$$\tau_3 \tfrac{d}{dt} x_3 = -a_3\,(x_3 - x_2) + a_4(u_2 - x_3); \quad y_3 = x_3$$

$$\text{where } f = a_0 \cdot x_1 \cdot \max[0, (x_2 - x_{2s})]$$

with x_1 for the reactant mass or concentration in the vessel content fluid where the reaction takes place, x_2 for the content temperature, x_3 for the jacket temperature (lumping both metal walls and heating/cooling fluid temperatures), u_2 for the entry temperature of the heating/cooling fluid from the supply subsystem to the jacket, and u_1 for the reactant feed flow.

Below the ignition temperature x_{2s}, f is zero. Above, f is proportional to the reactant concentration x_1 and the fluid contents temperature $x_2 - x_{2s}$.

The mass flow disappearing from the mass balance of reactant x_1 is set proportional to f (with coefficient a_1).

The thermal power production is also set proportional to f, with coefficient a_2. The exothermal reaction relates to $a_2 < 0$.
During production, the contents temperature x_2 must always stay above the ignition temperature x_{2s}, which calls for high closed loop performance. The inventory of reactant must also be closely controlled, for safety reasons.

The following numerical values are specified:

$$x_{2s} = 0.90;\ a_0 = \frac{1}{r_2 - x_{2s}} = 10.;$$
$$a_1 = 1.0;\ a_2 = -1.0;\ a_3 = 1.0;\ a_4 = 1.0;$$
$$\tau_1 = 0.20;\ \tau_2 = 1.00;\ \tau_3 = 0.20;$$

where a small τ_1/τ_2 signifies a relatively small inventory of reactant in the reactor, and where a large τ_3/τ_2 signifies a relatively large heat capacity of the jacket. The values for τ_1, τ_2, τ_3 imply a time scaling of typically $1000 \rightarrow 1$.

For the nominal steady state operating conditions at

$$\bar{x}_1 = 1.0;\ \bar{x}_2 = 1.0;\ \bar{x}_3 = 0.0;\ \text{that is}\ \bar{u}_2 = -1.0;\ \bar{u}_1 = 1.0;$$

the open loop poles are at

$$s_1 = +3.4640;\ s_2 = +0.7049;\ s_3 = -10.1734$$

The closed loop bandwidth for the temperature control in its linear range is specified at ≥ 15.0 rad/s.

The actuator stroke is saturated at

$$u_{2_{lo}} = -1.10 \text{ and } u_{2_{hi}} = +1.10$$

and the maximum actuator slew rate is

(a) $|du_2/dt| = 200.0$/s or (b) $|du_2/dt| = 5.0$/s

The proportional gain of the concentration controller R_1 is to be: $k_{p_1} \geq 10.0$

No slew saturations are specified for both $r_1(t)$ and $r_2(t)$.

The test sequence is defined as follows:

- start in closed loop operation with all initial conditions and inputs r_1, r_2 at zero,
- apply a reference step of size $r_2 = 1.0$; and wait for equilibration
- then add a load step of size r_1 up to max. stabilizable or to $r_1 = 1.00$, whichever is smaller.

The control targets are

- to attain the full equilibrium production at $r_1 = 1.0;\ r_2 = 1.0$ as fast as possible,
- and then control with the closed loop bandwidth specified above, with
  - an additional reactant inflow step $\Delta r_1 = +0.020$
  - and after equilibration a step change in a_2 of $\Delta a_2 = +0.020$
  - and after equilibration an additional temperature reference step $\Delta r_2 = -0.020$
- and fast shut-down of production by resetting first r_1 to zero, and then r_2 to zero.

□

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