

J. Pitman

# Combinatorial Stochastic Processes

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Author

Jim Pitman  
Department of Statistics  
University of California, Berkeley  
367 Evans Hall  
Berkeley, CA 94720-3860  
USA  
*e-mail: pitman@stat.Berkeley.edu*

Editor

Jean Picard  
Laboratoire de Mathématiques Appliquées  
UMR CNRS 6620  
Université Blaise Pascal (Clermont-Ferrand)  
63177 Aubière Cedex  
France  
*e-mail: jean.picard@math.univ-bpclermont.fr*

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## Foreword

Three series of lectures were given at the 32nd Probability Summer School in Saint-Flour (July 7–24, 2002), by the Professors Pitman, Tsirelson and Werner. The courses of Professors Tsirelson (“Scaling limit, noise, stability”) and Werner (“Random planar curves and Schramm-Loewner evolutions”) have been published in a previous issue of *Lectures Notes in Mathematics* (volume 1840). This volume contains the course “Combinatorial stochastic processes” of Professor Pitman. We cordially thank the author for his performance in Saint-Flour and for these notes.

76 participants have attended this school. 33 of them have given a short lecture. The lists of participants and of short lectures are enclosed at the end of the volume.

The Saint-Flour Probability Summer School was founded in 1971. Here are the references of Springer volumes which have been published prior to this one. All numbers refer to the *Lecture Notes in Mathematics* series, except S-50 which refers to volume 50 of the *Lecture Notes in Statistics* series.

1971: vol 307	1980: vol 929	1990: vol 1527	1998: vol 1738
1973: vol 390	1981: vol 976	1991: vol 1541	1999: vol 1781
1974: vol 480	1982: vol 1097	1992: vol 1581	2000: vol 1816
1975: vol 539	1983: vol 1117	1993: vol 1608	2001: vol 1837 & 1851
1976: vol 598	1984: vol 1180	1994: vol 1648	2002: vol 1840
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1978: vol 774	1988: vol 1427	1996: vol 1665	
1979: vol 876	1989: vol 1464	1997: vol 1717	

Further details can be found on the summer school web site  
<http://math.univ-bpclermont.fr/stflour/>

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*Jean Picard*

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