

Claudia Prévôt · Michael Röckner

# A Concise Course on Stochastic Partial Differential Equations

 Springer

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Library of Congress Control Number: 2007925694

Mathematics Subject Classification (2000): 35-XX, 60-XX

ISSN print edition: 0075-8434

ISSN electronic edition: 1617-9692

ISBN-10 3-540-70780-8 Springer Berlin Heidelberg New York

ISBN-13 978-3-540-70780-6 Springer Berlin Heidelberg New York

DOI 10.1007/978-3-540-70781-3

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Typesetting by the authors and SPi using a Springer L<sup>A</sup>T<sub>E</sub>X macro package

Cover design: WMXDesign GmbH, Heidelberg

Printed on acid-free paper SPIN: 11982159 VA41/3100/SPi 5 4 3 2 1 0

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