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Attractivity and Bifurcation for Nonautonomous Dynamical Systems

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To Professor Bernd Aulbach
and my parents

Preface

This book has been developed from my dissertation, which I wrote at the University of Augsburg from 2002 to 2005. I first became acquainted with several definitions of attractor for nonautonomous dynamical systems when I was preparing my diploma thesis, and the question arose whether a nonautonomous bifurcation theory can be founded based on suitable notions of nonautonomous attractor (and repeller).

At the beginning of my time as a Ph. D. student, I developed local notions of attractor and repeller for several time domains (the past, the future, the whole time and finite time intervals), and I distinguished between two bifurcation scenarios. The first scenario describes the loss of attractivity and repulsivity, and the second one deals with transitions of attractors and repellers. All definitions are introduced in Chapter 2 of this book. As a test for the new definitions, I then considered asymptotically autonomous differential equations; these are systems whose behavior becomes autonomous when time tends to the past or the future. I found conditions for the occurrence of a nonautonomous bifurcation in case the underlying autonomous system admits a bifurcation (see Chapter 7). Moreover, I developed nonautonomous counterparts for classical one-dimensional bifurcation patterns (see Chapter 6).

The remaining part of my work was focussed on the study of qualitative properties of the local notions of attractivity and repulsivity. I showed that these are suitable to describe the *global* asymptotic behavior via Morse decompositions (see Chapter 3), and for linear systems, I introduced notions of dichotomy and dichotomy spectra for the four different time domains (see Chapter 4). Furthermore, I constructed invariant manifolds of nonlinear systems for the different time domains in order to obtain attractivity and repulsivity from the linearization (see Chapter 5).

Writing this book would not have been possible without the aid of many people to whom I would like to express my gratitude. First of all, I would like to thank my supervisor Professor Bernd Aulbach, who unfortunately suddenly and unexpectedly passed away on January 14, 2005, at the age of 57 years. I am grateful for his longstanding support while writing my diploma thesis and dissertation. I benefited from his great ability to explain complicated facts very clearly and lucidly, and I am thankful to him for many fruitful discussions. Moreover, I am greatly indebted to Professor Fritz Colonius who became my advisor after the death of Professor Aulbach. He was very interested in the details of my work, and I was very encouraged by his positive attitude to my ideas and suggestions. Furthermore, I am grateful to Professor Lars Grüne for his interest in my work and for being a referee for my dissertation. I would also like to thank Dr. Stefan Siegmund for many useful discussions and remarks, especially in the first year of my work. Special thanks go to my friends and colleagues Dr. Christian Pötzsche and Dr. Ludwig Neidhart for reading the manuscript and making useful comments. I also thank the *Deutsche Forschungsgemeinschaft* for the financial support I received from them, when I was a member of the *Graduiertenkolleg "Nichtlineare Probleme in Analysis, Geometrie und Physik"* in the department for mathematics and physics at the University of Augsburg. Finally, I would like to thank my parents for making it possible for me to study mathematics and for their support during all these years.

Augsburg, February 2007 *Martin Rasmussen*

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