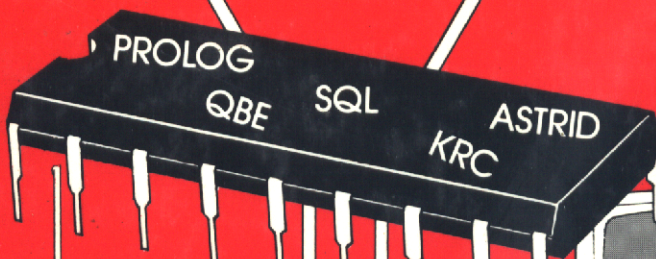


# Logic, Algebra and Databases

Peter Gray

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LOGIC, ALGEBRA AND DATABASES



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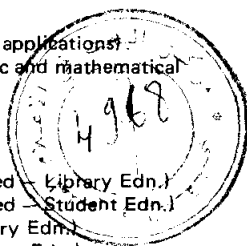
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