

Fundamentals
of
Engineering Elasticity
Second Edition

S. F. BORG

World Scientific

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157 2639

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Second Edition

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World Scientific

Singapore • New Jersey • London • Hong Kong

Published by

World Scientific Publishing Co. Pte. Ltd.

P O Box 128, Farrer Road, Singapore 9128

USA office: 687 Hartwell Street, Teaneck, NJ 07666

UK office: 73 Lynton Mead, Totteridge, London N20 8DH

Library of Congress Cataloging-in-Publication data is available.

Printed in 1962 by Litton Educational Pub., Inc.

FUNDAMENTALS OF ENGINEERING ELASTICITY

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ISBN 981-02-0164-8

981-02-0165-6 pbk



Printed in Singapore by JBW Printers & Binders Pte. Ltd.

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IN MEMORY OF AUDREY

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PREFACE TO THE FIRST EDITION

This book is intended for sophomore and junior students in engineering curricula.

It may be trite to repeat that we are living in an era of "exploding technology," but for engineers this statement does perhaps bear repetition. In any event, it is this fact which has led the author to write this textbook. It is his conviction that the properly educated engineer and engineering scientist of the present and future must have a grasp of the fundamentals of engineering.

If the modern engineering curriculum is to include all of the groundwork knowledge in the many different fields that the well-prepared engineering student must know, then it will be impossible to cover, in the four-year engineering curriculum, the subject material which in the past was included in structural engineering. This broad field included strength of materials (or mechanics of materials), elementary structural analysis, and elementary structural design.

How then are we to prepare the student for truly professional engineering practice in the present and future?

It would seem that there is only one method that will fit within the framework of a four-year undergraduate curriculum, and this is to present all subjects from their truly fundamental points of view. We must go back to the origins of all fields, re-exploring the assumptions, hypotheses and approximations that ultimately led to the development of the engineering forms of the various subjects.

In the subject now called "strength of materials," going back to the origins means taking as our starting point the subject matter of the mathematical theory of elasticity. Otherwise stated, the presentation of material in this text is based on the assumption that the fountainhead, or essential source, of all knowledge of structural theory and practice is the mathematical theory of elasticity.

The mathematical theory of elasticity may be called the parent, and the engineering elasticity (or strength of materials) the offspring, even though the latter developed at an earlier time than the former, because the mathematical and scientific justification of the theories developed in the strength of materials is only to be found in the body and extensions of the mathematical theory of elasticity. The engineering elasticity is essentially an approximation or simplification of the more exact theory, so that the practicing engineer could utilize the predictions and results of mathematical elasticity. Textbooks on strength of materials have presented the engineering form of the mathematical theory of elasticity, but rarely, if ever, has the relationship between parent and offspring been made clear. Understanding the relationship would seem to be essential if the student is to have

a good understanding of the coverage presented. To present the subject in such a manner and with such goals is what the present textbook attempts to do for the student

Because this book is written for the beginning student of structural engineering, we start with an explanatory chapter in which the basic philosophy behind the writing of the book is outlined. The purpose of the book is stated clearly and, as additional background, a brief historical summary of the field is presented. This will further help to explain to the student the purpose and intent of the text.

Tensor, the family name of the quantities of mathematical physics and, hence, of engineering, is a concept all modern engineers should understand and be familiar with. The tensor is introduced most naturally in this book because straightforward matrix-tensor statements are especially adaptable to discussions of engineering elasticity. For this reason, matrix-tensor arguments are utilized wherever possible. A basic, elementary treatment of matrix-tensor theory, suitable for our purposes, is given in Chapter 2. In this chapter also the elements of finite difference calculus are dealt with, because this also is a valuable tool that engineers should be familiar with and because elementary finite difference methods fit naturally into the framework of the coverage of the book.

The equations of the linearized theory of elasticity are presented next, and then it is shown how the ordinary, simplified structures of everyday engineering usage are analyzed using outgrowths and simplified forms of the basic, more exact theories. It is clearly shown just what approximations and assumptions are introduced in the basic theory of the engineering analysis — and it is shown why they are introduced.

In the remaining chapters the analysis of the key structural units (the bar, beam, etc.) is presented, first in the more exact linearized theory of elasticity solutions, and then in the approximate engineering solutions.

Extensions of the engineering analysis (the shear-moment curve construction, conjugate beam method, etc.) are introduced, where applicable, to indicate the directions in which this field has advanced.

Many problems dealing with the text material have been included, because an undergraduate engineering or science student can truly master his subject only when he can solve quantitative problems in connection with it.

In summary, then, the book has been written because the author believes the student in engineering must have a training and background in the field covered by the text. It is also the author's sincere hope that practicing engineers in the fields of applied mechanics and structural engineering will find the book worth while.

S. F. BORG
May 1961

PREFACE TO THE SECOND EDITION

The second edition follows the format of the first one. Several typographical errors have been corrected and several new topics or extensions of the original material have been included in an Appendix following Chapter 13 at the end of the book.

Overall however, the two fundamental premises have been adhered to, namely:

1. In order to attain a real understanding of the subject which we call "strength of materials" or "mechanics of materials" one must go back to the beginnings of these fields — the linearized mathematical theory of elasticity. Hence the title of the book stressing the words *engineering elasticity*.
2. The field of engineering elasticity is a good one to utilize in introducing to the undergraduate engineering student the important and useful topic of tensors. And for the engineer the matrix representation of the tensor is the easiest to visualize and to understand. Hence the use of the matrix-tensor notation.

The author wishes to express sincere thanks to his publishers, World Scientific Publishing Co., for reprinting the text and for their continual help and encouragement in seeing this task through to completion.

S. F. BORG
March 3, 1990

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