M. Rusinowitch J. L. Rémy (Eds.)

Conditional Term Rewriting Systems

Third International Workshop, CTRS-92
Pont-à-Mousson, France, July 8-10, 1992
Proceedings

Springer-Verlag
Berlin Heidelberg New York
London Paris Tokyo
Hong Kong Barcelona
Budapest

Series Editors

Gerhard Goos
Universität Karlsruhe
Postfach 69 80
Vincenz-Priessnitz-Straße 1
W-7500 Karlsruhe, FRG

Juris Hartmanis
Cornell University
Department of Computer Science
4130 Upson Hall
Ithaca, NY 14853, USA

Volume Editors

Michaël Rusinowitch
Jean-Luc Rémy
CRIN & INRIA-Lorraine
B. P. 239, F-54506 Vandoeuvre-Les-Nancy Cedex, France

CR Subject Classification (1991): F.4.1-2, D.3.1, I.2.3

ISBN 3-540-56393-8 Springer-Verlag Berlin Heidelberg New York
ISBN 0-387-56393-8 Springer-Verlag New York Berlin Heidelberg

Printed in Germany

Typesetting: Camera ready by author
45/3140-543210 - Printed on acid-free paper

To the memory of Stéphane Kaplan

Preface

This volume contains the papers presented at the Third International Workshop on Conditional Term Rewriting Systems held on 8-10 July 1992 at Pont-à-Mousson, France.

The first CTRS workshop was held in 1987 at the Université of Paris XI in Orsay and the second one took place in 1990 at Concordia University in Montreal. The proceedings have been published in the Lecture Notes in Computer Science, Springer Verlag, volume 308 and volume 516 respectively. Topics of these workshops include conditional rewriting and its applications to programming languages, specification languages, automated deduction, constrained rewriting, typed rewriting, higher-order rewriting, graph-rewriting.

We would like to thank the *Région Lorraine*, the *Centre de Recherche en Informatique de Nancy* (which is the joint computer science laboratory of the Universities of Nancy and the *Centre National de la Recherche Scientifique*) and the Lorraine research center of the *Institut National de la Recherche en Informatique et Automatique* for their support.

We are also grateful to Christiane Guyot and Véronique Poirel for their help with many tasks during the preparation of the workshop.

September 1992 — The Organizers,

Michaël Rusinowitch
Jean-Luc Rémy

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ALGEBRAIC SEMANTICS OF REWRITING TERMS AND TYPES

Karl Meinke
Department of Mathematics and Computer Science,
University College Swansea,
Singleton Park,
Swansea SA2 8PP,
Great Britain.
June 1992

Abstract We present a universal algebraic framework for rewriting terms and types over an arbitrary equational specification of types and typed combinators. Equational type specifications and their initial algebra semantics were introduced in Meinke [1991b]. For an arbitrary equational type specification (ε, E) we prove that the corresponding rewriting relation $\overset{R(\varepsilon, E)^*}{\longrightarrow}$ coincides with the provability relation $(\varepsilon, E) \vdash$ for the equational calculus of terms and types. Using completeness results for this calculus we deduce that rewriting for ground terms and ground types coincides with calculation in the initial model $I(\varepsilon, E)$ of the equational type specification.

0. INTRODUCTION

The basic theory of universal algebra in higher types was introduced in Poigné [1986], Möller [1987], Möller et al [1988] and Meinke [1990], [1992]. A higher type universal algebra is a many–sorted universal algebra in which the carrier sets have some type theoretic structure. Applications of the theory of higher type universal algebra to programming language definition and hardware verification are discussed in Meinke [1991a] and Meinke and Steggles [1992]. The basic theory of higher type universal algebra concerns only the propositional type constructions $\times$ and $\rightarrow$ for product and function spaces. In Meinke [1991b] a more general algebraic theory of abstract type constructions and typed combinator systems was introduced, and an equational specification theory for algebras of types and combinators was presented, including results on initial models and completeness of equation and type assignment calculi. Within this general framework, complex type constructions such as predicative and impredicative Π and Σ types and recursive types may be modelled algebraically. The universality of the theory allows for user defined type disciplines and the specification of abstract data types with a rich type structure.

An equational type specification is a pair $Spec = ((\Gamma, \varepsilon), (\Sigma, E))$ consisting of a type signature Γ, and equational theory ε of types, a combinator signature Σ and an equational theory E of combinators. The model class $Mod(Spec)$ forms a category which admits an initial algebra $I(Spec)$ which we may take as the initial algebra semantics of $Spec$. From the viewpoint of computation it is natural to consider term rewriting as a mechanism for computation in the initial model $I(Spec)$. In this paper we introduce a theory of term rewriting for equational type specifications. An important new feature of this theory is that both rewriting of types and rewriting of terms are allowed. Given a combinator term t over a specification $Spec = ((\Gamma, \varepsilon), (\Sigma, E))$ the oriented equations of ε allow us to rewrite type terms in t while the oriented equations of E allow us to rewrite combinator terms in t. Our main result is a Correspondence Theorem for rewriting which establishes that the rewrite relation $\xrightarrow{R(\varepsilon, E)^*}$ on combinator terms using the oriented equations of ε and E coincides with the provability relation $(\varepsilon, E) \vdash$ in the equational calculus of terms and types. By a completeness theorem for this calculus it follows that rewriting coincides with calculation in the initial model $I(Spec)$.

The structure of this paper is as follows. In section 1 we present essential prerequisites from universal algebra and category theory. In section 2 we review basic definitions and results on the algebraic theory of types and combinators. In section 3 we introduce the fundamental definitions and concepts of rewriting for types and combinators. In section 4 we review the equational calculus of terms and types. Finally in section 5 we prove a Correspondence Theorem for rewriting and equational deduction. We have attempted to make the paper self contained. However the reader may wish to consult Meinke [1991b] and Meinke and Wagner [1992] for more detailed proofs and examples.

1. ALGEBRAIC AND CATEGORY THEORETIC PRELIMINARIES

The notation for many–sorted universal algebra that we use is taken from Meinke and Tucker [1992] where all of the basic technical definitions and results on algebra that we require may be found. Let us make our notation explicit.

An *S–sorted signature* Σ consists of a non–empty set S, the elements of which are *sorts*, and an $S^* \times S$–indexed family $\langle\, \Sigma_{w,\,s} \mid w \in S^*,\, s \in S \,\rangle$ of sets (where S^* denotes the set of all words over S). For the empty word $\lambda \in S^*$ and any sort $s \in S$, each element $c \in \Sigma_{\lambda,\,s}$ is a *constant symbol* of sort s; for each non–empty word $w = s(1) \ldots s(n) \in S^+$ and any sort $s \in S$, each element $\sigma \in \Sigma_{w,\,s}$ is a *function symbol* of *domain type* w, *codomain type* s and *arity* n. Thus we can define Σ to be the pair $(S, \langle\, \Sigma_{w,\,s} \mid w \in S^*,\, s \in S \,\rangle)$.

Let Σ be an S–sorted signature. An *S–sorted Σ algebra* is an ordered pair (A, Σ^A), consisting of an S–indexed family $A = \langle\, A_s \mid s \in S \,\rangle$ of carrier sets A_s and an $S^* \times S$ indexed family $\Sigma^A = \langle\, \Sigma^A_{w,s} \mid w \in S^*,\, s \in S \,\rangle$ of sets of constants and algebraic operations. For each sort $s \in S$, $\Sigma^A_{\lambda,s} = \{\, c_A \mid c \in \Sigma_{\lambda,s} \,\}$, where $c_A \in A_s$ is a constant that interprets c. For each $w = s(1) \ldots s(n) \in S^+$ and each sort $s \in S$, $\Sigma^A_{w,s} = \{\, f_A \mid f \in \Sigma_{w,s} \,\}$, where $f_A : A^w \to A_s$ is an operation with domain $A^w = A_{s(1)} \times \ldots \times A_{s(n)}$ and codomain A_s which interprets f. As usual, we let A denote both a Σ algebra and its S–indexed family of carrier sets.

We require one essential construction from category theory, the cofibration construction, which we use to structure the model class of an equational type specification.

1.1. Definition. Let C be a category and *Cat* be a category of categories.