# Geometric Reasoning for Perception and Action

Workshop Grenoble, France, September 16-17, 1991 Selected Papers



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## Preface

Geometry is a powerful tool to solve a great number of problems in robotics and computer vision. Impressive results have been obtained in these fields in the last decade. However addressing problems of the actual world requires reasoning about uncertainty and complex motion constraints by combining geometric, kinematic, and dynamic characteristics. Dealing with such characteristics is an attribute of intelligence and autonomy. Uncertainty generally implies that it is necessary to "see to act" (as in control) and to "act to see" (as in perception). It also has strong consequences for the planning process, since appropriate "motion strategies" have to be applied: for instance, how can a blind and inaccurate robot progress towards the exit of a given maze using its own proximity sensors for guiding its motions? Dealing with complex motion constraints clearly requires devising new models and new search techniques, since the nature of the problem is intrinsically different from traditional path planning which only deals with non-collision constraints and simple integrable kinematic constraints. Even if some interesting results have been recently obtained, motion planning with non-holonomic kinematic constraints and/or dynamic constraints is still an open question. It is a new challenge in robotics and vision to try to solving these problems and integrate, whenever necessary, appropriate planning, sensing and control techniques. A necessary step towards the achievement of such an ambitious goal is to develop appropriate geometric reasoning techniques with reasonable computational complexity.

One of the activities of the French Joint Research Programme in Artificial Intelligence (PRC-IA) is to study how geometric reasoning can contribute to solving these problems. A workshop on this topic was held at LIFIA-IRIMAG in Grenoble (France) on September 16-17, 1991. This workshop was jointly organized by LIFIA-IRIMAG and PRC-IA. It was funded by the French Ministry of Research and Space (MRE), the French Ministry of Education (MEN) and LIFIA-IRIMAG. The scientific programme consisted of 18 contributions, 15 of which were selected for inclusion in the present book.

The selected contributions cover several important areas in the field of robotics and computer vision. Part 1 deals with motion planning with kinematic and dynamic constraints. Part 2 investigates motion planning and control in the presence of uncertainty, and presents planning based techniques, reactivity, and visual servoing methods. Part 3 addresses geometric problems related to visual perception. Part 4 deals with numerical problems linked to the implementation of practical algorithms for visual perception.

Impressive results were presented during the workshop in both the fields of motion planning and visual perception. However, there is obviously a lack of joint results in these fields. Hopefully the purpose of several new European joint projects is to study action-perception-control interaction problems. A special issue of the International Journal of Robotics Research will be dedicated to this topic in the near future.

Finally I would like to thank all the participants who contributed to the success of the workshop and the quality of the present volume.

Grenoble, June 1993

Christian LAUGIER.

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# Motion Planning under Kinematic and Dynamic Constraints

# **BIBLIOTHEQUE DU CERIST**

# Shortest Paths of Bounded Curvature in the Plane

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Abstract. Given two oriented points in the plane, we determine and compute the shortest paths of bounded curvature joigning them. This problem has been solved recently, by Dubins in the no-cusp case, and by Reeds and Shepp otherwise. We propose a new solution based on the minimum principle of Pontryagin. Our approach simplifies the proofs and makes clear the global or local nature of the results.

### 1 Introduction

The question considered here is the following : given two oriented points  $(M_i, \theta_i)$ and  $(M_f, \theta_f)$  in the plane, determine and compute the shortest piecewise regular paths joining them, along which the curvature is everywhere bounded by a given  $\frac{1}{R} > 0$ . Minimizing the length is meaningful both in the class of paths which are  $C^1$  and piecewise  $C^2$ , and in the slightly larger class of paths admitting a finite number of cusps.

This question appears in many applications : for instance Markov[3] studied the no-cusp case for joining pieces of railways. A 3-dimensional version applies to planning plumbary networks, or the version with cusps to any car driver.

Even without obstacles, characterizing the shortest paths is not simple. This was only done recently, by Dubins[2] in the no-cusp case, and by Reeds and Shepp[5] otherwise.

Our way of solving the question here is entirely different from theirs. It is both much simpler and better adapted to further generalization to the case of obstacles limiting moves. The essential tool we use is the powerful result of optimal control theory known as the "minimum principle of Pontryagin". We recall in Section 2 its basic version which we will use, and refer to classical books in control theory, like [1], [4], and [7], for its quite delicate proof.

In Section 3 we apply the principle to our case, and deduce some general crucial lemmas. Section 4 is devoted to the no-cusp case, and Section 5 to the more difficult case with cusps.

Our results are essentially the same as those of [2] and [5]. The interest of the present work lies in the method of proof, both simplified by the use of a single idea, and as local as the statements will allow. Indeed, we make a clear distinction between local and global proofs, and we insist on local proofs in view of further work dealing with obstacles.

Related results to be reported in a forthcoming publication have been obtained independently by Sussmann and Tang [6]. The results in [2] and [5] are also deduced from the minimum principle and new lights on the piecewise regularity of optimal controls.

### 2 The minimum principle : a basic version

Given are :

- two integers n and r, two points  $(x^i)$  and  $(x^f)$  in  $IR^r$ , and a compact subset U of  $IR^n$ ,
- a  $C^0$  function  $f(x, u) : IR^n \times U \to IR^n$ ,
- a  $C^0$  function  $f_0(x, u) : IR^n \times U \to IR^n$ .

A "control" is a piecewise continuous (but not necessarily continuous) function  $u(t): [0,T] \to U$ , for some T > 0.  $\mathcal{U}$  denotes the set of controls. We want to find a  $u \in \mathcal{U}$  which minimizes the integral

$$J(u) = \int_0^T f_0(x(t), u(t)) dt$$

Here  $x(t) = (x_1(t) \cdots, x_n(t)) : [0, T] \rightarrow IR^n$  is a solution of the differential system with boundary conditions

$$\begin{cases} \frac{dx_j}{dt} = f_j(x, u(t)) \ (j = 1, ..., n) \\ x(0) = x^i; \ x(T) = x^j \end{cases}$$
(1)

J(u) is called the "cost" of the "path" x(t), solution of (1), given the "control" u(t). The solution x(t) of the system with initial conditions  $x(0) = x^i$  is well determined for a given u. This is the case even in a much more general setting (see e.g. [7, Theorem II.4.11]). We denote by  $\mathcal{U}_i^f$  the subset of "admissible" controls  $u \in \mathcal{U}$  such that the associated paths x satisfy also the final conditions  $x(T) = x^f$ . The optimal control problem is to find controls  $* \in \mathcal{U}_i^f$  satisfying

$$J(\star) = \min_{u \in \mathcal{U}_I} J(u)$$

Such an \* is called "optimal", as well as the associated path x. Note that T is here arbitrary. But in the particular case  $f_0 \equiv 1$ , J(u) = T and we want to minimize the "time" to go from  $x^i$  to  $x^f$ .