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# Lecture Notes in Computer Science

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Rolf Klein

Concrete and Abstract  
Voronoi Diagrams

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# Preface

The Voronoi diagram is one of the most useful data structures in computational geometry, with applications in many areas of science. In this book, a unifying approach is proposed by introducing abstract Voronoi diagrams. These are not based on the notion of distance but rather on the concept of bisecting curves.

This book is a revised version of my Habilitationsschrift *On a generalization of planar Voronoi diagrams* that was accepted by the Faculty of Mathematics, University of Freiburg, FRG, in February 1989. I have included some recent developments, and have brought up to date the selection of open problems in the concluding remarks.

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Essen, October 1989

Rolf Klein

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