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FUNCTIONAL AND MULTI-VALUED DEPENDENCIES  
IN A RELATIONAL DATABASE AND THE THEORY  
OF BOOLEAN SWITCHING FUNCTIONS

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RR N° 142

Novembre 1978

Abstract

It is shown that the isomorphism between functional dependencies (FD's) in a relational schema and Boolean inference statements can be extended to handle multivalued dependencies (MVD's) as well. The Boolean structure for these dependencies is useful for several reasons :

- (1) it is concise, giving a better intuitive and manipulative grasp on FD's, MVD's and decompositions, and
- (2) it is effective in designing algorithms for making dependency inferences.

New algorithms for FD/MVD closure, cover and membership are presented as an application.

Key Words : Functional dependencies, multivalued dependencies, Boolean functions, schema decomposition, dependency closure, dependency cover, dependency membership.

Résumé

L'objectif de ce papier est de montrer que l'isomorphisme entre les dépendances fonctionnelles (FDs) d'un schéma relationnel et les fonctions booléennes peut être étendu au cas des dépendances multivaluées (MVDs). La structure de la fonction booléenne associée à ces dépendances est utile pour les raisons suivantes :

- (1) elle est concise et permet une meilleure compréhension des manipulations sur les FDs, MVDs et les décompositions, et
- (2) elle est efficace pour la conception d'algorithmes de couverture et de fermeture d'un ensemble de dépendances regroupant les FDs et les MVDs.

De plus, un algorithme permettant de tester si une dépendance appartient à la fermeture est proposé.

Mots-clés : Dépendances fonctionnelles, dépendances multivaluées, fonctions booléennes, décomposition d'un schéma, fermeture des dépendances, couverture des dépendances.

## 1. Introduction

Since the papers of Codd appeared in the early 1970's a great deal of attention has been focussed on the semantics of the relational data model, and in particular on the behavior of functional dependencies. This work has clarified the issues for standard dependencies, culminating in a number of papers appearing last year ([Fag 77b], [Ris 77], [BFH 77], [ABU 77]) which systematize the inferential structure of ordinary functional and multivalued dependencies, and demonstrate their relationship with lossless database decomposition.

However, although these theoretical developments now rest soundly established, good algorithms for manipulating multivalued dependencies (constructing inferences, closure, etc.) have not yet been found. A number of algorithms for functional dependencies are known ([DC 73], [BB 76], [Fag 77a], etc), and they are used frequently in database design procedures ([DC 73], [Ber 76], [DP 78]) but these algorithms have not been generalized for the case where multivalued dependencies are also present. The clumsiness of known manipulation methods is perhaps one reason why few database design procedures handling these MVD's have been developed — the authors know of only one well-specified algorithm [ZM 78], although Beerl is credited with one in [BFH 77] and Fagin sketches another in [Fag 77c].

This paper shows the somewhat surprising result that the isomorphism between functional dependencies and Boolean inference statements can be extended to encompass multivalued dependencies as well. The implications of this statement are manifold : first, a better understanding of the way dependency statements interact is obtained. The Boolean representation of a complicated inference from a set of dependency statements is often extremely concise, permitting immediate understanding of what is going on, while the dependency equivalent is a mess ; and one appreciates better the way MVD's interact with FD's. Second, a better understanding of the decomposition results in [Fag 77b], [Ris 77], and [ABU 77] is a by-product. Third, of course, one is led to efficient algorithms for manipulating sets of dependencies which contain MVD's.