

COMPUTER ENGINEERING SERIES

Advanced Graph Theory and Combinatorics

Michel Rigo

BIBLIOTHEQUE DU CERIST



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