## An Efficient Model for a Camera Behind a Parallel Refractive Slab

Aless Lasaruk<sup>1</sup> · Tomas Pajdla<sup>2</sup>

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## Abstract

We present a new and efficient solution for the forward world to image projection of a pinhole camera with distortions placed behind a planar refractive slab. Firstly, we introduce a novel way to compute the projection by reducing the problem to finding a real quantity called a slab shift. We characterize a physically meaningful slab shift as the unique solution of a fixed point equation on the one hand and as a specific uniquely defined root of a quartic polynomial in the unknown slab shift on the other. In the latter case we obtain a closed-form formula, which provides the unique physically meaningful projection. Secondly, we develop an approximation of the projection through the slab that reaches single-precision floating point accuracy for practically relevant problem instances with considerably lower computational costs compared to the exact solution. We demonstrate the accuracy and the efficiency of our method with realistic synthetic experiments. We demonstrate with real experiments that our method enables efficient camera calibration behind the windshield for automotive industry applications.

Keywords Camera model · Calibration · Windshield · Automotive · Open source

## **1** Introduction

A refractive medium placed in front of a camera influences the optical perception of the scene by the camera and so introduces a geometric distortion of the resulting image. Therefore, efficient models for geometric camera projection behind refractive mediums are important for applications in the vehicle, naval, and aviation industries. Such applications include among many others autonomous driving (Kukkala et al., 2018; Hanel & Stilla, 2018), underwater vehicles (Jordt-Sedlazeck & Koch, 2013; Kawahara et al., 2013), and aerial spatial geometry reconstruction (Schönberger & Frahm, 2016) respectively.

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 Aless Lasaruk aless.lasaruk@zf.com
Tomas Pajdla pajdla@cvut.cz

<sup>1</sup> ZF Friedrichshafen, Friedrichshafen, Germany

<sup>2</sup> Czech Institute of Informatics, Robotics and Cybernetics, Czech Technical University in Prague, Prague 6, Czechia

In many applications the cameras are mounted close to a thin refractive medium with smooth interfaces. For example, the cameras in a vehicle are often mounted directly behind the windshield in the mirror cover. While the refractive interfaces of the medium may not be flat as a whole in such a case, the camera observes the scene only through a very small part of the medium. Consequently, the dominant effect of the geometric image distortion introduced by the medium originates not necessarily from the curvature of the interfaces but rather from the local properties of the medium such as the average refraction index, average thickness of the medium, and the orientation of the interface tangents with respect to the optics. The above properties can be modeled well by a planar slab approximation of the medium. Therefore, the geometric projection of a camera behind a planar refractive medium is an interesting approximation model for the above mentioned practical applications.

A common requirement for computer vision methods in automotive applications is the ability to implement the methods on embedded hardware. Examples are image rectification and stitching (Gao et al., 2018), stereo matching (Juárez et al., 2016), real-time surround-view visualization, structure reconstruction (Hanel et al., 2016), and camera calibration behind a windshield of a vehicle (Hanel & Stilla, 2018). A way to achieve efficiency is to devise specialized geometric models of the sensor. One approach is to

