An Updated Survey of GA-Based Multiobjective Optimization Techniques

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After using evolutionary techniques for single-objective optimization during more than two decades, the incorporation of more than one objective in the fitness function has finally become a popular area of research. As a consequence, many new evolutionary-based approaches and variations of existing techniques have recently been published in the technical literature. The purpose of this paper is to summarize and organize the information on these current approaches, emphasizing the importance of analyzing the operations research techniques in which most of them are based, in an attempt to motivate researchers to look into these mathematical programming approaches for new ways of exploiting the search capabilities of evolutionary algorithms. Furthermore, a summary of the main algorithms behind these approaches is provided, together with a brief criticism that includes their advantages and disadvantages, degree of applicability, and some known applications. Finally, future trends in this area and some possible paths for further research are also addressed.

Categories and Subject Descriptors: I.2.8 [**Artificial Intelligence**]: Problem Solving, Control Methods, and Search—*Heuristic methods*

General Terms: Algorithms

Additional Key Words and Phrases: Artificial intelligence, genetic algorithms, multicriteria optimization, multiobjective optimization, vector optimization

1. INTRODUCTION

Multiobjective optimization is without a doubt a very important research topic both for scientists and engineers, not only because of the multiobjective nature of most real-world problems but also because there are still many open questions in this area. In operations research, more than 20 techniques have been developed over the years to try to deal with functions that have multiple objectives, and many approaches have been suggested, going all the way from naively combining objectives into one, to the use of game theory to coordinate the relative importance of each objective. The fuzziness of this area lies in the fact that there is no accepted definition of "optimum" as in single-objective optimization. Hence it is difficult to even compare the results of one method to another's because, normally, the decision about

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the "best" answer corresponds to the so-called (human) *decision maker*.

There have been other surveys of multiobjective optimization techniques in the mathematical programming literature, from which the papers by Cohon and Marks [1975]; Hwang et al. [1980]; Stadler [1984]; Lieberman [1991]; Evans [1984]; Fishburn [1978]; and Boychuk and Ovchinnikov [1973] are probably the most comprehensive. The most remarkable survey of multiobjective optimization in the evolutionary computing literature is the one by Fonseca and Fleming [1994; 1995c]. However, little detail is provided in this work on how each method works, only a few applications of each technique are given, and their corresponding operations research roots are scarcely mentioned. Furthermore, several other approaches have arisen since the publication of Fonseca's paper, and the intention of the present work is to provide researchers and students with an updated survey that can be used (to a certain extent) as a self-contained document for anyone with a previous (at least basic) general knowledge of genetic algorithms (GAs). Those who need additional information about genetic algorithms should refer to Goldberg [1989]; Holland [1992]; Michalewicz [1992]; and Mitchell [1996].

This paper emphasizes the importance of looking at previous work in operations research, not only to get a full understanding of some of the existing techniques, but also to motivate the development of new GA-based approaches. Finally, applications of each method are also given to provide the reader with a more complete idea of the range of applicability and the underlying motivation of each of these techniques. A brief criticism appears after the description of each technique listing its advantages, possible drawbacks and limitations, and (in some cases) possible ways to exploit its characteristics or even improve performance.

2. STATEMENT OF THE PROBLEM

Multiobjective optimization (also called multicriteria optimization, multiperformance, or vector optimization) can be defined as the problem of finding

> *"a vector of decision variables which satisfies constraints and optimizes a vector function whose elements represent the objective functions. These functions form a mathematical description of performance criteria which are usually in conflict with each other. Hence, the term "optimize" means finding such a solution which would give the values of all the objective functions acceptable to the designer."* [Osyczka 1985]

Formally, we can state this as follows: Find the vector $\bar{x}^* = [x_1^*, x_2^*, \ldots, x_n^*]^T$ that will satisfy the m inequality constraints:

$$g_i(\bar{x}) \geq 0 \quad i = 1, 2, \ldots, m \tag{1}$$

the p equality constraints