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A common generalization of Chvátal–Erdős' and Fraisse's sufficient conditions for hamiltonian graphs

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Abstract

Let $G = (V, E)$ be a k-connected graph of order n. For $S \subset V$, let $N(S)$ be its neighborhood set and let $J(S) = \{u \notin S \mid N(u) \supseteq S\}$ if $|S| \geqslant 2$ and $J(S) = \emptyset$ otherwise. If there exists some s, $1 \leqslant s \leqslant k$, such that every independent set X of $s+1$ vertices has a vertex u satisfying $|N(X \setminus \{u\})| + |N(u) \cup J(X \setminus \{u\})| \geqslant n$, then G is hamiltonian. From this main theorem, we derive new sufficient conditions for hamiltonian graphs. Some of these results are improvements and/or generalizations of various known results. In particular, sufficient conditions of Ore (1960), Chvátal and Erdős (1972), Fraisse (1986) and E. Flandrin et al. (1991) are easily derived.

Keywords: Hamiltonian cycle; Hamiltonian path; Neighborhood union; Neighborhood intersection

1. Introduction

We consider only finite simple graphs $G = (V, E)$. For $S \subset V$, let $N_i(S) = \{u \in V \mid d(u, S) = i\}$ where $d(u, S)$ is the distance from u to the nearest vertex in S. For simplicity we adopt the notation $N(S)$ if $i = 1$. Let $NC(G) = \min\{|N(u) \cup N(v)| \mid uv \notin E\}$ if G is noncomplete and $NC(G) = n-1$ otherwise and let $\sigma_p = \min\{\sum_{i=1}^{p} d(v_i) \mid \{v_1, \ldots, v_p\}$ is an independent set$\}$. Instead of $\sigma_1(G)$ we use the more common notation $\delta(G)$. For an independent set $S \subset V(G)$ of $s+1$ vertices we define the $s+1$ *neighborhood intersections* $S_i = \{v \in V(G) \setminus S \mid |N(v) \cap S| = i\}$, $1 \leqslant i \leqslant s+1$. The cardinalities of S_i will be denoted by λ_i. If S contains at least two vertices we denote $n - \sum_{i>2} |N_i(S)|$ by $n(S)$. Also let $I(S) = \{u \notin S \mid |N(u) \cap S| \geqslant \max\{2, |S|-1\}\}$, $J(S) = \{u \notin S \mid N(u) \supseteq S\}$. If $|S| < 2$ we set $J(S) = \emptyset$. Obviously $n(S) \leqslant n$ and $J(S) \subseteq I(S)$ unless $|S| = 2$ in which case $J(S) = I(S)$.

Degree conditions have long been fundamental tools in the study of hamiltonian properties. During the last decade, several results dealing with various neighborhood conditions were obtained. In Sections 2 and 4, new sufficient conditions for the existence of hamiltonian cycles are given. Some of these results strengthen and

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