Available at
www.ComputerScienceWeb.com
POWERED BY SCIENCE @DIRECT

Information Processing Letters 85 (2003) 307–315

Information Processing Letters

www.elsevier.com/locate/ipl

A combinatorial algorithm for MAX CSP

Mayur Datar [1], Tomás Feder, Aristides Gionis [*,1], Rajeev Motwani [2], Rina Panigrahy

Department of Computer Science, Gates 4B, Stanford University, Stanford, CA 94305-9045, USA

Received 13 December 2001; received in revised form 13 September 2002

Communicated by S. Albers

Abstract

We consider the problem MAX CSP over multi-valued domains with variables ranging over sets of size $s_i \leqslant s$ and constraints involving $k_j \leqslant k$ variables. We study two algorithms with approximation ratios A and B, respectively, so we obtain a solution with approximation ratio $\max(A, B)$.

The first algorithm is based on the linear programming algorithm of Serna, Trevisan, and Xhafa [Proc. 15th Annual Symp. on Theoret. Aspects of Comput. Sci., 1998, pp. 488–498] and gives ratio A which is bounded below by s^{1-k}. For $k = 2$, our bound in terms of the individual set sizes is the minimum over all constraints involving two variables of $(1/2\sqrt{s_1} + 1/2\sqrt{s_2})^2$, where s_1 and s_2 are the set sizes for the two variables.

We then give a simple combinatorial algorithm which has approximation ratio B, with $B > A/\mathrm{e}$. The bound is greater than s^{1-k}/e in general, and greater than $s^{1-k}(1-(s-1)/2(k-1))$ for $s \leqslant k-1$, thus close to the s^{1-k} linear programming bound for large k. For $k = 2$, the bound is $\frac{4}{9}$ if $s = 2$, $1/2(s-1)$ if $s \geqslant 3$, and in general greater than the minimum of $1/4s_1 + 1/4s_2$ over constraints with set sizes s_1 and s_2, thus within a factor of two of the linear programming bound.

For the case of $k = 2$ and $s = 2$ we prove an integrality gap of $\frac{4}{9}(1 + \mathrm{O}(n^{-1/2}))$. This shows that our analysis is tight for any method that uses the linear programming upper bound.

Keywords: Algorithmical approximation; Analysis of algorithms; Combinatorial problems; Databases; Design of algorithms; Graph algorithms

1. Introduction

We consider the problem MAX CSP (Maximization Constraint Satisfaction Problem) over multi-valued domains [12]. An instance of this problem consists of n variables x_i, where each variable takes values from a corresponding set S_i. Each set S_i has $s_i = |S_i|$ values, and we define s to be the largest set size, i.e., $\max_i s_i$. The problem statement also specifies a set of m constraints, where each constraint is defined in the following way: For the jth constraint, a set of indices $A_j \subseteq [n]$ specifies which variables

[*] Corresponding author.

E-mail addresses: datar@cs.stanford.edu (M. Datar), tomas@theory.stanford.edu (T. Feder), gionis@cs.stanford.edu (A. Gionis), rajeev@cs.stanford.edu (R. Motwani), rinap@cisco.com (R. Panigrahy).

[1] Supported by NSF Grant IIS-0118173 and a Microsoft Research Graduate Fellowship.

[2] Supported by NSF Grant IIS-0118173 and an Okawa Foundation Research Grant.

0020-0190/02/$ – see front matter
PII: S0020-0190(02)00435-0