A Batch Processing Constant Modulus Algorithm

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Abstract—We present a batch processing constant modulus algorithm (BP-CMA) derived by a nonlinear optimiration approach to minimizing the constant modulus (CM) criterion. BP-CMA is a line search iteration algorithm. The search direction may be taken as deepest descent direction or Newton direction. The exact step size is obtained from the roots of a cubic equation. The initial value is calculated by using the eigenvectors of the signal subspace. The BP-CMA with the Newton direction has a fast convergence rate and can converge to the minima of the CM criterion after a few iterations.

Index Terms—Batch processing algorithm, constant modulus algorithm, signal-subspace.

I. INTRODUCTION

MINIMIZATION of the constant modulus (CM) criterion has been widely studied for blind channel equalization and blind source separation (see [1] and the references therein). The most popular way to minimize the CM criterion is to use a stochastic gradient based constant modulus algorithm (CMA). However, the CMA has a slow convergence rate and is sensitive to the selection of the step size and initial value.

In this letter, we present a batch processing CMA (BP-CMA) derived by using a nonlinear optimization approach. BP-CMA is a line search iteration algorithm, which involves three factors: search direction, step sire and initial value. The two widely used search directions are the steepest descent direction and the Newton direction. We will focus on determining the step size and initial value. The step size is exactly obtained from the roots of a cubic equation. The initial value is calculated by using the eigenvectors of signal subspace. BP-CMA converges quickly. The BP-CMA with the Newton direction can converge to the minima of CM criterion after a few iterations. Note that the Newton type method based CMA has been considered earlier (see, e.g., [2]). However, the step sire and initial value issues were not addressed.

II. CONSTANT MODULUS ESTIMATORS

Consider a generic data model given by

$$\mathbf{y}(n) = \sum_{k=1}^{K} \mathbf{h}_k s_k(n) + \mathbf{w}(n) \stackrel{\mathrm{def}}{=} \mathbf{H}\mathbf{s}(n) + \mathbf{w}(n) \qquad (1)$$

where $\mathbf{s}(n) = [s_1(n) \ \cdots \ s_K(n)]^T$ is the vector of K input signals (K known). $\mathbf{y}(n)$ is the output vector of dimension P. $\mathbf{h}_k$ are the length-P channel vectors, $\mathbf{H} = [\mathbf{h}_1 \ \cdots \ \mathbf{h}_K]$ is the $P \times K$ channel matrix, and $\mathbf{w}(n)$ is a noise vector. The hypotheses on the model are given as follows.

H1) $s_k(n)$ are zero-mean i.i.d. sub-Gaussian sequences with variance $E\{|s_k(n)|^2\} = \sigma_s^2$, i.e., the kurtosis $K(s_k(n)) = E\{|s_k(n)|^4\} - 2E^2\{|s_k(n)|^2\} - |E\{s_k(n)^2\}|^2 < 0$, and if $s_k(n)$ are complex-valued, $E\{s_k(n)^2\} = 0$, which means that they are circularly symmetric.

H2) The noise $\mathbf{w}(n)$ is a zero-mean Gaussian vector with covariance matrix $\sigma_w^2 \mathbf{I}_P$. Moreover, if $\mathbf{w}(n)$ is complex-valued, then it is also circularly symmetric.

H3) The channel matrix $\mathbf{H}$ is of full column rank.

Let a length-P vector $\mathbf{g}$ denote a linear estimator and $z(n) = \mathbf{g}^H \mathbf{y}(n)$ be the output of the estimator. The CM estimator we consider minimizes the following CM criterion:

$$\mathbf{g}_{cm} = \arg\min J(\mathbf{g}) \qquad (2)$$

where $J(\mathbf{g}) = E\{(|z(n)|^2 - \gamma)^2\}$ is the CM criterion and γ is a positive design parameter known as the dispersion constant.

Proposition 1: The extremes of the function $J(\mathbf{g})$ are within the signal subspace spanned by the column vectors of channel matrix $\mathbf{H}$.

Proposition 2: In the absence of noise (i.e., $\sigma_w = 0$) or when the columns of the channel matrix are orthogonal to each other (i.e., $\mathbf{H}^H\mathbf{H} = \alpha \mathbf{I}_K, \alpha > 0$), the minima of the function $J(\mathbf{g})$ satisfy the zero-forcing (ZF) conditions.

Remarks: Proposition 1 was established in [3]. Proposition 2 was established in [4], [5]. Proposition 2 shows that in these two special cases, the CM estimator is equivalent to a ZF estimator.

III. BATCH PROCESSING CMA

In this section, we present our BP-CMA derived by a line search iteration algorithm. In the line search strategy, the algorithm chooses a direction $\mathbf{p}$, and searches along this direction from the current iteration to the next iteration with a lower value of the cost function. The iteration is given by (see [6])

$$\mathbf{g}_{i+1} = \mathbf{g}_i + \mu_i \mathbf{p}_i \qquad (3)$$

where the positive scalar μ_i is the step sire, which can be found by solving the following one-dimensional minimization problem:

$$\mu_i = \arg\min_{\mu>0} J(\mathbf{g}_i + \mu \mathbf{p}_i). \qquad (4)$$

The effectiveness of a line search method depends on the appropriate choices of both the direction $\mathbf{p}_i$ and the step sire μ_i. Using a good initial value $\mathbf{g}_0$ can improve the steady-state performance

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