PERGAMON

Nonlinear Analysis 38 (1999) 827–867

Nonlinear Analysis

Local T-sets and degenerate quasilinear elliptic bilateral problems with an L^1-datum

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Received 24 July 1996; accepted 23 October 1997

Keywords: T-sets; Nonlinear elliptic bilateral problems; Degenerate inequations; Renormalized solutions; L^1-datum; Uniqueness and stability theorems

1. Introduction

Let $p>1$ be a real number, Ω an arbitrary open subset of $\mathbb{R}^N$, and S a compact subset of $\overline{\Omega}$ whose N-dimensional Lebesgue measure is zero; in the sequel, S will be called singularity. Let a and b be two nonnegative functions on $\overline{\Omega}$ which might degenerate (i.e., vanish or go to infinity) on S. Let ψ and Ψ be two measurable functions on Ω with $\psi \le \Psi$ a.e. in Ω and $\Psi \in L^\infty(\Omega)$. Put

$$K(\psi,\Psi) = \{v:\Omega \to \mathbb{R} \text{ measurable } \mid \psi \le v \le \Psi \text{ a.e. in } \Omega\}$$

and

$$\mathscr{A}u \doteq -\mathrm{div}(\hat{a}(x,u,\nabla u)) + b(x)|u|^{\gamma-1}u \qquad (1)$$

where $\hat{a}:\Omega\times\mathbb{R}\times\mathbb{R}^N \to \mathbb{R}^N$ is a Carathéodory function such that $\hat{a}/a$ satisfies the general conditions of Leray–Lions [15] and γ a real number with $p-1<\gamma<(N/(N-p))(p-1)$ if $1<p<N$ and $p-1<\gamma<\infty$ otherwise.

We will study the questions of existence and uniqueness of functions u belonging to $K(\psi,\Psi)$ satisfying, in a sense which will be precised later, the inequality

$$\mathscr{A}u \ge \mu \quad \text{on } K(\psi,\Psi) \qquad (2)$$

where μ is a given function in $L^1(\Omega)$.

In the nondegenerate case, for Ω bounded and $p>p_{c_0}=2-1/N$, unilateral problems associated with quasilinear operators and irregular data were considered by many authors, cf., for instance, Boccardo and Gallouët [9], for the p-Laplacian and $L^1(\Omega)$-data,

0362-546X/99/$ - see front matter
PII: S0362-546X(98)00122-9