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Option: Mathématiques Appliquées

Par

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THEME

Stabilisation uniforme de quelques équations aux dérivées partielles à coefficients variables.

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BIBLIOTHEQUE DU CERIST

Uniform decay rates of certain classes of Partial Differential Equations with variable coefficients .

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2008/2009

Dédicace

BIBLIOTHEQUE DU CERIST

À mes parents, mes frères et ma sœur.

À mon mari, mes enfants: Dhia Eddine, Kawther et Romaissa.

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