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THEME

**Robustness of infinite dimensional
stochastic systems**

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Abstract

The subject of this thesis is to study the problems of robust stability and robust stabilization for linear deterministic systems on real Hilbert spaces which are subjected to Lipschitzian stochastic structured multi-perturbations. within the framework of stability radii.

First we consider the case where the operators describing the structure of the perturbations are bounded. We establish characterizations of the stability radius in terms of a Lyapunov equation and the corresponding inequalities. These characterizations are used to obtain a computational formula for this radius.

Then, we study the problem of maximizing the stability radius by state feedback. We establish conditions for the existence of suboptimal controllers in terms of a Riccati equation. We showed also how the supremal stability radius can be determined in terms of this equation.

Finally, we investigate the robustness of stability in the case where the operators structure are unbounded. We show how we can generalize the results established in the bounded case for this case. We characterize the stability radius in terms of a Lyapunov equation similar to the one used in the bounded case. These characterizations enable us to determine a lower bound for the stability radius.

Key words: Wiener process, Stochastic differential equation, Exponential stability, Mean square stability, Robustness, Stability radius, Lyapunov equation, Riccati equation.

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