BIBLIOTHEQUE DU CERIST

UNIVERSITY OF CALIFORNIA
RIVERSIDE

Completely Non-Boundary Normal Operators in Finitely Connected Domains

A Dissertation submitted in partial satisfaction
of the requirements for the degree of

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in

Mathematics

by

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ABSTRACT OF THE DISSERTATION

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In this dissertation, we study completely non-boundary normal (c.n.∂n) operators on finitely connected domains. Thus, let X be a finitely connected domain, i.e., a bounded connected open subset of $\mathbb{C}$ whose boundary, ∂X, consists of $n+1$ nonintersecting, analytic, Jordan curves. Boundary normal (∂-normal) operators are normal operators on a Hilbert space K, with spectrum in ∂X. We are interested in operators of the form $T = PN|H$ that satisfy:

(i) $\sigma(T) \subset X$,

(ii) N a ∂-normal operator in $\mathcal{L}(K)$, $K \supset H$,

(iii) $f(T) = Pf(N)|H$, for all f in $R(X)$,

(iv) there is no nonzero reducing subspace on which (T) is ∂-normal.

P is the orthogonal projection of K onto H, and $R(X)$ is the algebra of rational functions with poles off $\bar{X}$. Our theorem is: Let T be a c.n.∂n operator, with minimal normal dilation N. Then, the spectrum of N is absolutely continuous.

Then we define the $\mathcal{C}_0$ class and study special cases of $\mathcal{C}_0$ c.n.∂n operators, namely, the Sarason Operators. Thus, let Z be the *shift* operator on $H^2(\partial X)$ and u an inner function in $H^\infty(X)$. We show that the commutant of a Sarason operator, $S(u) = PZ|H^2(\partial X) \ominus uH^2(\partial X)$, can be lifted to the commutant of Z. This extends a theorem of Sarason to finitely connected domains. To do that, we extend and use Nehari's theorem in our setting.

Finally, we extend Page's theorem, when the underlying Hilbert space is finite dimensional, to finitely connected domains. Page's theorem is the vector valued version of Nehari's theorem.

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