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par

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Temps de Passage de Paquets d'Ondes de Basses Fréquences ou Limités en Bandes de Fréquences par une Barrière de Potentiel.

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Contents

Barrier Traversal Time for Low Frequency or Band Limited Dispersive Wave Packets.

In this thesis we study a transmission problem for the one dimensional Klein-Gordon equation on a finite interval coupled with the wave equation on the rest of the real axis. This corresponds to a model of a particle in quantum mechanics moving in a world of one space dimension with a potential barrier. The particle will present the tunnel effect, i.e. its appearance in places which are classically prohibited. Our main aim is the study of the dynamics of this phenomenon.

We find a phenomenon of advanced transmission of certain types of waves packets while crossing the barrier. This means that aspects of the incident wave packet can be found on the other side before the arrival of a test signal which propagates without the presence of the potential barrier. We consider two cases: modulated Gaussians or functions which are limited in a frequency band. In both cases, classical causality is not violated, because the admissible initial conditions can be more or less located, but cannot have compact support.

This is in accordance with physical experiments carried out by A. Enders and G. Nimtz in 1992 for microwaves and R.Y. Chiao, P.G. Kwiat and A.M. Steinberg in 1993 for optical photons. A theoretical approach was proposed by J.M. Deutch and F.E. Low in 1993 for Gaussians using the Laplace transform in time, confirming the zero traversal time for this type of initial conditions.

In this thesis we first adapt the approach of Deutch and Low to treat Gaussians which are modulated by functions having a general form, to approach the question of advanced transmission of information. Then we treat barriers of slightly perturbed form. Finally we use another approach, that of the spectral theory (following the lines of J. Weidmann, 1987 and E. Croc/Y. Dermenjian, 1995) which allows us to define wave packets which are limited in frequency bands. This corresponds to the notion of the experimenters to send wave packets having a main frequency. Here we take as a starting point the paper on delayed reflection by F. Ali Mehmeti and V. Régnier (published in 2004). We find that for sufficiently large barriers the traversal time varies from almost zero for low frequency packets to the time needed by a test signal, when the frequency tends to infinity.

We thus have refined in several directions the results on the dynamics of the tunnel effect.

Key words: Klein-Gordon equation, wave equation, advanced transmission, transmission coefficient, reflection coefficient, frequency band, rectangular potential barrier, tunnel effect.