

Multi-Box Splines: Analysis and Examples

By
Abdellatif Bettayeb

Doctor of Philosophy
Division of Mathematics
University of Dundee
Dundee
September 2007

Contents

1	Introduction	2
2	Preliminaries and Definitions	6
2.1	Introduction	6
2.2	Background	6
2.3	Uniform B-Spline	9
2.3.1	Properties	10
2.4	Box splines	11
2.4.1	Properties	11
2.4.2	Examples of Box splines	12
2.5	Multi-Box splines	16
2.5.1	Definitions and properties	16
2.5.2	Examples of Multi-box splines	22
2.5.3	Example 1 [10]	22
2.5.4	Example 2 [9]	23
2.5.5	Example 3 [9]	24
3	Multi-Box Splines Techniques	25
3.1	Fourier transform	25
3.1.1	Definition	25
3.1.2	Properties	26
3.1.3	Example 1	27
3.1.4	Example 2	28
3.1.5	Example 3	28
3.1.6	Example 4	29
3.1.7	Example 5	29
3.1.8	Example 6	30
3.1.9	Example 7	30

3.2	Subdivision and Refinement	31
3.2.1	Definition	31
3.2.2	Subdivision Algorithm and Example	33
4	Some Piecewise Constant Splines	36
4.1	Background	36
4.2	Proof of Theorem 4.1.1	36
4.3	General Results	40
4.4	Examples	43
4.5	Properties	49
5	Multi-box Splines Properties	55
5.1	Introduction	55
5.2	Properties of local generator ψ_n	56
5.3	Recursive construction of a local generator	58
5.4	Linear independence and shifts	60
5.5	Multi-box splines and B-splines	61
5.6	Stability of generators and linear independence of shifts	67
6	Some Quadratic Multi-Box Splines	72
6.1	Background	72
6.2	First Function	76
6.3	Second Function	81
6.3.1	Condition 1: $\widehat{g}_2(u, v) = \widehat{g}_2(u, -v)$	81
6.3.2	Condition 2: $\widehat{g}_2(u, v) = -\widehat{g}_2(-u, v)$	85
6.4	Third Function	88
6.4.1	Condition 1: $\widehat{g}_3(u, v) = \widehat{g}_3(-u, v)$	88
6.4.2	Condition 2: $\widehat{g}_3(u, -v) = -\widehat{g}_3(u, v)$	91
6.5	Conclusion	93
7	Some Cubic Multi-Box Splines	96
7.1	Background	96
7.2	First Function	100
7.3	Second Function	104
7.4	Conclusion	106

8	Some Linear Multi-Box Splines on a Hexagonal Mesh	108
8.1	Background	108
8.2	First Function	114
8.3	Second Function	121
8.3.1	Condition 1: $\hat{g}_2(u, v) = \hat{g}_2(u, -v)$	121
8.3.2	Condition 2: $\hat{g}_2(u, v) = e^{-iu} \hat{g}_2(-u, v)$	123
8.4	Third Function	126
8.5	Fourth Function	126
8.6	Conclusion	127
9	Some Linear Multi-Box Splines on a Unknown support	134
9.1	Background	134
9.2	The Constructing Algorithm	136
9.3	Constructing the Linear Multi-box spline ϕ	137
10	Conclusions	157