**Mathematics Study Resources 4** 

# **Olaf Manz**

# Encrypt, Sign, Attack

A compact introduction to cryptography



## Mathematics Study Resources

#### **Series Editors**

Kolja Knauer Departament de Matemàtiques Informàtic Universitat de Barcelona Barcelona, Barcelona, Spain

Elijah Liflyand Department of Mathematics Bar-Ilan University Ramat-Gan, Israel This series comprises direct translations of successful foreign language titles, especially from the German language.

Powered by advances in automated translation, these books draw on global teaching excellence to provide students and lecturers with diverse materials for teaching and study.

Olaf Manz

# Encrypt, Sign, Attack

A compact introduction to cryptography



Olaf Manz Worms, Germany

 ISSN 2731-3824
 ISSN 2731-3832 (electronic)

 Mathematics Study Resources
 ISBN 978-3-662-66015-7 (eBook)

 https://doi.org/10.1007/978-3-662-66015-7
 (eBook)

© The Editor(s) (if applicable) and The Author(s), under exclusive licence to Springer-Verlag GmbH, DE, part of Springer Nature 2022

Translation from the German language edition: "Verschlüsseln, Signieren, Angreifen" by Olaf Manz, © Springer-Verlag GmbH Deutschland, ein Teil von Springer Nature 2019. Published by Springer Berlin Heidelberg. All Rights Reserved.

This work is subject to copyright. All rights are solely and exclusively licensed by the Publisher, whether the whole or part of the material is concerned, specifically the rights of translation, reprinting, reuse of illustrations, recitation, broadcasting, reproduction on microfilms or in any other physical way, and transmission or information storage and retrieval, electronic adaptation, computer software, or by similar or dissimilar methodology now known or hereafter developed.

The use of general descriptive names, registered names, trademarks, service marks, etc. in this publication does not imply, even in the absence of a specific statement, that such names are exempt from the relevant protective laws and regulations and therefore free for general use.

The publisher, the authors, and the editors are safe to assume that the advice and information in this book are believed to be true and accurate at the date of publication. Neither the publisher nor the authors or the editors give a warranty, expressed or implied, with respect to the material contained herein or for any errors or omissions that may have been made. The publisher remains neutral with regard to jurisdictional claims in published maps and institutional affiliations.

This Springer imprint is published by the registered company Springer-Verlag GmbH, DE, part of Springer Nature. The registered company address is: Heidelberger Platz 3, 14197 Berlin, Germany

### Preface

Have you ever wondered whether mobile phones can be used to confide even the most secret secrets? Or whether online banking is really secure these days? Or whether an electronic signature on contracts sent by e-mail meets legal requirements? All of this has something to do with the **encryption** – or **ciphering** – of data, which is sent or stored on data carriers every day in large and ever-increasing quantities via data highways or "wireless".

Textbooks and reference books take a more scientific approach to the topic of data encryption under the title of **cryptography.** They deal with the mathematical theories of the common procedures, describe their algorithms and program-technical realizations, and also deal with many topics of the organizational implementation. As a basis for lectures or seminars, it must in the first instance be the goal to introduce students to scientific work and to introduce them to areas of current research. Practitioners working in the subject also need a correspondingly comprehensive presentation. On the other hand, there are also numerous popular science publications that aim at a generally understandable level. This works very well in this case, since simple ciphering methods can easily be brought to the attention of interested laymen and can be substantiated with examples from everyday practice. The mathematics behind it, however, usually remains hidden.

This book aims to be a balancing act between the two. It is a fact that cryptography can be understood quite comprehensively with very little mathematics. Our goal is therefore, without a theoretical superstructure, to deal specifically with the most important procedures of **encryption**, **signing** and **authentication**, and to present them in a compact and mathematically understandable manner, which is reflected in many practical examples.

• We focus first on symmetric ciphers, where anyone who knows the cipher procedure can decode it. The procedures go back to antiquity to the **Caesar cipher**, in which each letter in the alphabet is replaced by the letter three places further down. The **Vigenère cipher** from the sixteenth century does this much more subtly, while more modern methods such as the **Triple DES** (Data Encryption Standard) and especially today's standard method **AES** (Advanced Encryption Standard) are considerably more complex.

- But how is it supposed to work that you can encrypt but can't decrypt even with the help of the biggest and most modern computers? The keyword is **public key.** We will learn about the standard methods: **RSA** relies on the difficulty of decomposing large natural numbers into factors, and **Diffie-Hellman** and **ElGamal** exploit the problem that "discrete logarithms" cannot be computed efficiently enough. Here, we even run into "elliptic curves" with **ECDH**.
- Ciphering would certainly be unnecessary if there were not rogues and especially also
  professional attackers who would expect political, military or economic advantage
  from the knowledge of secret data and therefore try to "crack" the encryption. In addition to classical attacks using statistical analysis and Friedman's coincidence index,
  we learn about Pollard's methods for effectively factorizing large natural numbers to
  potentially "crack" RSA. Finally, we also attack the "discrete logarithm" with BabyStep-Giant-Step and Pohlig-Hellman.
- Particularly fatal, however, is an attack in which an unauthorized person not only passively listens in but also actively engages in the message traffic and changes it in their own way. In this case, the recipient of a message is completely unaware of whether the information received in this form really originates from exactly the sender specified. In order to prevent this situation, **digital signatures** are used, for example the **RSA**, **DSA** or **ECDSA** procedure, thus giving a **man-in-the-middle attack** no chance.
- Of course, we will always deal with practical applications. Historically interesting are, for example, the Illuminati cipher and the Enigma machine. The Internet with HTTPS is perhaps the most prominent modern application for secure data transmission, but wireless WLAN networks and the Bluetooth radio interface are also well protected today. The PGP Pretty Good Privacy method is widely used for e-mails, while mobile communications with GSM are only partially secure against eavesdropping, but those with UMTS/LTE are much more secure. Another focus is on online banking, credit cards and Bitcoins. Finally, e-passports with their biometric data are also designed to be forgery-proof. Last but not least, data stored on hard disks, and thus passwords in particular, must be protected against unauthorized access.

The target audience for this book is basically anyone who is enthusiastic about the topic; in particular, it is also intended as an introduction to more advanced literature. We will have to do relatively little, but nevertheless some mathematics. We will need arithmetic with binary numbers (bits) and with remainders modulo a natural number, as well as an understanding of permutations, both for the conceptual background and for one or the other formal derivations. However, we will build this up piece by piece, with special emphasis on the plausibility of the relationships. So, let's plunge into the adventure – and have fun.

As a guide, here is a brief reading guide for the four chapters of this book in advance:

- Chapter 1 is intended as a "warm up", with an overview of important historical ciphers.
- Chapter 2 examines **symmetric ciphers S** (standard methods: **Triple-DES** and **AES**). These cipher procedures depend on a parameter to be kept secret, the so-called **key k**, with the help of which decryption can also be performed. Thus, participant **T** encrypts a secret message **m** as follows:
  - S(m, k).
- But how does participant **T** deliver the comparatively short key **k** to the authorized recipient of the secret message **m**, also by secret means? Chapter 3 introduces the concept of **public-key ciphers E** (standard methods **RSA**, (**EC**)**DH**, **ElGamal**). In these methods, which however require much more computation time than symmetric ciphers, one cannot decrypt **e** from the knowledge of their **key** alone. Participant **T** therefore encrypts **k** using **E** and sends the concatenation as a whole:
  - $\ E(k,\,e) \| \ S(m,\,k).$
- But wait: How can the recipient be sure that the received message really comes from participant T in exactly this form? In Chap. 4, this problem is solved by means of digital signatures sig (standard methods RSA, (EC)DSA). Participant T signs the message m, more precisely a digital fingerprint h(m) of m (standard procedure SHA), and additionally sends the signature sig(h(m)) in the following concatenation:
  - $E(k, e) \parallel S(m \parallel sig(h(m)), k).$
- Sometimes, however, a more conventional method, the checksum MAC, is used as an alternative to the digital signature (standard procedures CBC-MAC, HMAC):
   E(k, e)|| S(m || MAC(m), k).
- Before sending a secure message, participant T usually has to log on to a system (e.g., mobile phone, computer network, bank server) and legitimize himself. As explained at the end of Chap. 4, this can be done "classically", for example by entering a PIN or password, but also with the aid of modern **public key** procedures and in particular with a **digital signature**:
  - $\mathbf{T} \triangleright \ldots \mathbf{E}(\mathbf{k}, \mathbf{e}) \parallel \mathbf{S}(\mathbf{m} \parallel \mathbf{sig}(\mathbf{h}(\mathbf{m})), \mathbf{k}).$

Worms, Germany

Olaf Manz

## Contents

1	Bas	ics and History	1		
	1.1	What It Is About: The Scenario	1		
	1.2	Alphabets and Digitisation	3		
	1.3	Caesar Cipher	6		
	1.4	Secret Writing of the Illuminati	8		
	1.5	Vigenère Cipher	10		
	1.6	Kasiski and Friedman Attack	12		
	1.7	Enigma Machine	15		
2	Symmetric Ciphers				
	2.1	Keys and Attack Strategies.	19		
	2.2	Vernam Cipher and Pseudo-Randomness	22		
	2.3	GSM Mobile Communications	24		
	2.4	Feistel Cipher	26		
	2.5	Data Encryption Standard DES	29		
	2.6	Operating Modes of Block Ciphers	38		
	2.7	UMTS/LTE Mobile Communications and Digital Television	41		
	2.8	Advanced Encryption Standard AES	43		
	2.9	Hard Disk and ZIP Archive	49		
3	Public-Key Ciphers				
	3.1	Factorization and RSA Cipher	53		
	3.2	Internet and WLAN	59		
	3.3	Monte Carlo Prime Numbers	61		
	3.4	Attack by Factorization	65		
	3.5	Discrete Logarithm and Diffie-Hellman	69		
	3.6	Attack with Baby and Giant Steps	74		
	3.7	Bluetooth and ECDH	78		
	3.8	ElGamal Cipher	83		
	3.9	Knapsack and Merkle-Hellman Cipher	85		

4	Digi	tal Signature	87	
	4.1	Man-in-the-Middle Attack and Authentication	87	
	4.2	RSA and ElGamal Signature	90	
	4.3	Hash Value and Secure Hash Algorithm SHA	94	
	4.4	Email with PGP and WhatsApp	99	
	4.5	DSA and ECDSA Signature	102	
	4.6	Online Banking	107	
	4.7	Blind Signature and Cryptocurrencies	110	
	4.8	Password Security and Challenge Response	114	
	4.9	Mobile Phone, Credit Card and Passport	118	
Bib	Bibliography			